Midterm

Nov 18, 2002. Closed book. No calculators allowed. Problem 1-7: 10 points each. Problem 8: 16 points. Problem 9: 14 points.

(1): Show that if $f(x)$ is differentiable at x_0 then $f(x)$ is continuous at x_0.

(2): Find the $y'(x)$ and $y''(x)$ at $(x, y) = (1, 1)$ where $y(x)$ is implicitly defined by $x^2+y^4 = 2$.

(3): Find the limits of the following expressions, distinguish between the cases when the limit is ∞ or $-\infty$ and the limit does not exist.

- **(a):** $\lim_{x\to 0}(\ln \frac{1}{x})^x$
- **(b):** $\lim_{x\to\infty}(x - x^2 \ln(1 + \frac{1}{x}))$

(4): Find dy/dx where $y = a^{x^x} + x^{a^x} + x^{x^a}$ and $x > 0$, $a > 0$.

(5): Show how Newton's method can be used to find $\sqrt[3]{2}$. (State the procedure but need not give the numerical value).

(6): Let f be a real valued function defined on $\{x \geq 0\}$ satisfying

- **(a):** $f(0) = -1$,
- **(b):** $f'(x) \geq 1/2$ for all $x \geq 0$.

Prove that $f(x) = 0$ has exactly one solution on $\{x \geq 0\}$.

(7): Graph $y = e^{-x^2/2}$ over the interval $|x| \leq 3$, indicate critical points and inflection points, and determine whether the critical points are local extremums.

(8): True or False? Give reason for your answer.

- **(a):** $\ln(x) = o(x^{0.1})$ as $x \to \infty$,
- **(b):** $\sin(x) = o(\sinh(x))$ as $x \to 0$,
- **(c):** If $f(x) = O(g(x))$ and $g(x) = O(h(x))$ as $x \to \infty$, then $f(x) = O(h(x))$ as $x \to \infty$.
- **(d):** If $f(x) = O(h(x))$ and $g(x) = O(h(x))$ as $x \to \infty$, then $h(x) = O(g(x))$ as $x \to 0$.

(9): $f(x) = \exp(-1/|x|)$ for $x \neq 0$ and $f(0)$ is not defined.

- **(a):** Show that f has a removable singularity at 0.
- **(b):** With the singularity removed, f is now a function defined for all real x. Is f differentiable at 0? Explain.