Final Exam

Tue June 17, 2003, 6:00 PM. Do all the problems. Total 120 pts. Your final grade is the total points you get $\times 10/12$. SHOW ALL YOUR WORK.

1. (5 pts) Find $\partial z/\partial x$ and $\partial z/\partial y$ at $(6,3,2)$ where $z(x,y)$ is given implicitly by
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1=0$$

2. (10 pts) By about how much will
$$f(x,y,z)=e^x\cos yz$$
change as the point $P(x,y,z)$ moves from the origin a distance of $ds=0.1$ unit in the direction of $(1,1,1)/\sqrt{3}$?

3. (15 pts) The derivative of $f(x,y,z)$ at a point $P=(1,0,0)$ is greatest in the direction of $(1,-1,1)/\sqrt{3}$. In this direction the value of the derivative is 2
   (a) What is the derivative in the direction of $(1,1,-1)/\sqrt{3}$?
   (b) Find the tangent plane of the level surface at P.
   (c) Give an example of f satisfying the description above.

4. (10 pts) State (need not prove) the second derivative test for $f(x,y)$. You may assume that f and all its first and second derivatives are continuous in an open region R.

5. (15 pts) Use Lagrangian multipliers (ONLY) to solve the following problems.
   (a) (5 pts) Find the nearest points from origin on the curve $x^2+xy+y^2=2$.
   (b) (10 pts) Find the extreme values of the function $f(x,y,z)=xy+z^2$ on the circle in which the plane $y-x=0$ intersects the sphere $x^2+y^2+z^2=4$.

6. (15 pts) Switch the order of integration of
$$\int_0^1\int_{-1}^0\int_0^{y^2} dzdydx$$
to
   (a) $dydzdx$
   (b) $dydxdz$
   (c) $dxdydz$

   You don't need to find the numerical value of the integral.

7. (10 pts) Convert the following into spherical integral (need not evaluate)

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} dzdydx$$

8. (10 pts) Find the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

9. (20 pts)

   (a) (10 pts) State the Green's Theorem both in Normal and Tangential Forms. You may assume M, N and all their first derivatives are continuous in a region R and its boundary C.

   (b) (10 pts) Verify both of them on the region $R=\{x^2+y^2/4\leq 1\}$ with $M(x,y)=x$, $N(x,y)=-3y$. In other words, you have to evaluate the line integral and double integral independently for the Normal form, verify they are the same and repeat the process for the Tangential form.

10. (10 pts) Find the integral of $G(x,y,z)=x$ over the surface given by $\{y=x^2,\ 0\leq x\leq 2,\ 0\leq z\leq 3\}$