Final Exam

Jan 09 2003, 10:10 AM - 12:00 AM
Show all your work

(1) (10 pts) Compute $\frac{d}{dx}\exp(\int_x^{x^2-1}\tan^{-1}(y)dy)$.

(2) (35 pts)

(a): Compute the area and perimeter of the region enclosed by $y = x$ and $y = (x-2)^2$.

(b): Compute the volume obtained by rotating (a) around the x axis.

(c): Compute the surface area of (b).

(3) (15 pts) Which of the following improper integrals converge and which of them diverge? Explain.

(a): $\int_1^\infty \frac{1}{\sqrt{x^3+1}}dx$

(b): $\int_1^\infty \frac{\ln x}{x}dx$

(c): $\int_{-\infty}^\infty \frac{1}{\cosh x}dx$

(4) (10 pts each, max 60 pts) Find the following integrals.

(a): $\int_{-1}^1 \frac{1}{x^2+2x+3}dx$.

(b): $\int \frac{1}{1-x^4}dx$

(c): $\int \frac{\cot x}{1+\sin^2(x)}dx$

(d): $\int_0^{\pi/2} \frac{1}{1+\sin x}dx$

(e): $\int \sin^n x dx, \qquad n \in N$

(f): $\int_0^\infty x^n e^{-x}dx, \qquad n \in N$

(g): $\int \frac{1}{(1+x^2)^2}dx$