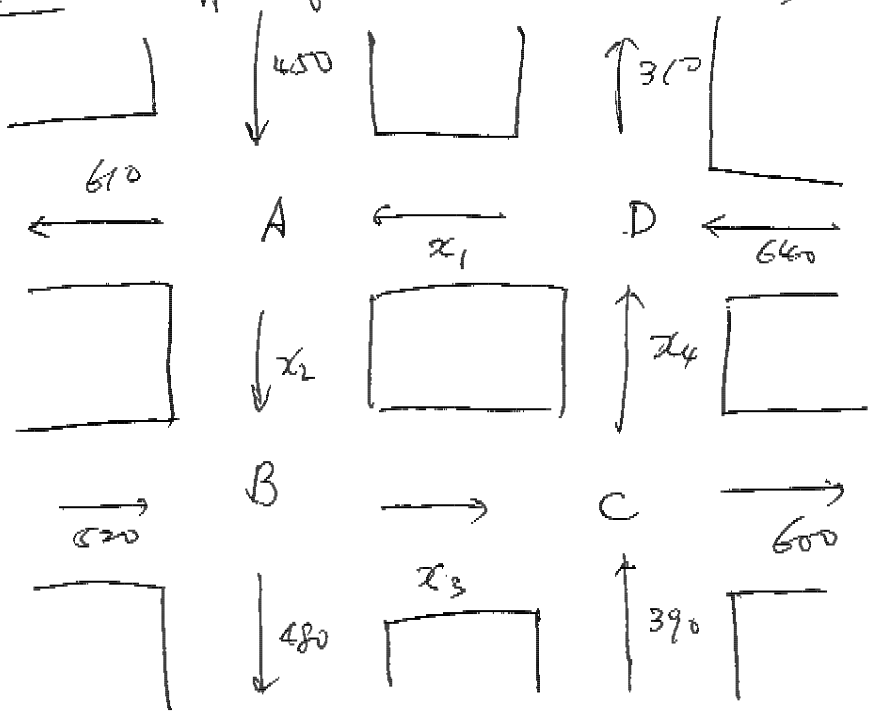
To motivate the study of systems of linear equations, we start by a few examples.

Example 1 traffic flow (Application 1)



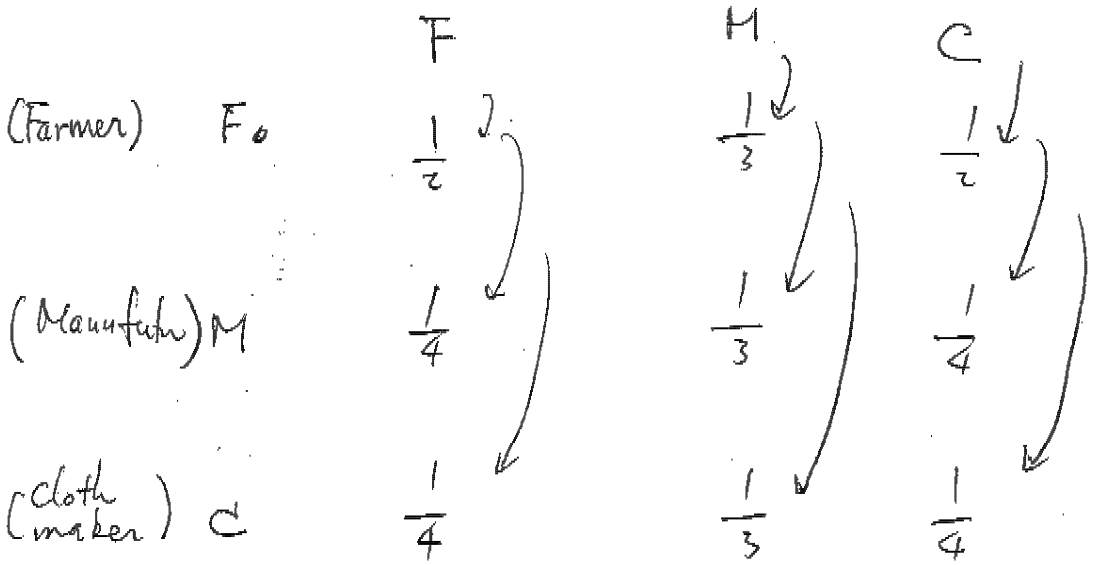
$$x_1 + 450 = x_2 + 610 \quad (A: \text{total in} = \text{total out})$$
$$x_2 + 520 = x_3 + 480 \quad (B)$$
$$x_3 + 390 = x_4 + 600 \quad (C)$$
$$x_4 + 640 = x_1 + 310 \quad (D)$$

Remark: Total number of equations
= total number of unknowns

But there is no unique solutions.

Why? (Hint: circulation)

Example 2 (Application 4)



(Trade Product by Product)

Question: If use currency, what is the fair Price of each Product?

$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{2}x_3 = x_1$$ (Total trade In = Total trade Out for "F")

$$\frac{1}{4}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = x_2$$ (Same for M)

$$\frac{1}{4}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = x_3$$ (for C)

Answer: $x_1 : x_2 : x_3 = 5 : 3 : 3$

(Why not unique solution?)

~~Aufgabe~~

Gauss elimination

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 3x_1 - x_2 - 3x_3 &= -1 \quad \downarrow \times(-3) \\ 2x_1 + 3x_2 + x_3 &= 4 \quad \downarrow \times(2) \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ -7x_2 - 6x_3 &= -10 \\ -x_2 - x_3 &= -2 \quad \times(-\tfrac{1}{7}) \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ -7x_2 - 6x_3 &= -10 \\ -\tfrac{1}{7}x_3 &= -\tfrac{4}{7} \end{aligned}$$

$$\Rightarrow \quad x_3 = 4, \quad x_2 = -2, \quad x_1 = 3$$

Augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 3 & -1 & -3 & -1 \\ 2 & 3 & 1 & 4 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & -1 & -1 & -2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & 0 & -\frac{1}{7} & -\frac{4}{7} \end{array}\right)$$

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  To get started, select "MATLAB Help" from the Help menu.

>> A = [ 1/2 1/3 1/2 ; 1/4 1/3 1/4 ; 1/4 1/3 1/4 ]

A =

    0.5000    0.3333    0.5000
    0.2500    0.3333    0.2500
    0.2500    0.3333    0.2500

>> x = [ 1 ; 1 ; 1]

x =

     1
     1
     1

>> n = 1; A^n*x

ans =

    1.3333
    0.8333
    0.8333

>> n = 3; A^n*x

ans =

    1.3634
    0.8183
    0.8183

>> n = 5; A^n*x

ans =

    1.3636
    0.8182
    0.8182

>> n = 100; A^n*x

ans =

    1.3636
    0.8182
    0.8182

>> ans/0.8182

ans =

    1.6666
    1.0000
    1.0000

>> exit
```