Sample problems for midterm (99/4/30, 8:00–9:50 AM)

(1) Write down the Taylor series of $\tan x$ around $x = \pi/4$ up to 3rd order.

(2) what is root test? give some examples

(3) For what values of α is

$$\sum_{n=N}^{\infty} n^{\alpha} \quad \text{convergent?}$$

If convergent, how does it depend on N?

(4) $$f(x,y) = \begin{cases} \dfrac{x^2-y^2}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Is f (i) continuous? (ii) differentiable? (iii) Does $D_{(\cos\theta, \sin\theta)} f(0,0)$ exist?

(iv) Does f have a local max or local min?

(5) What is the largest volume of the cube (with sides parallel to x, y, z axis) contained in $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$?

(6) Find the approximate solution of $\dfrac{2+\sqrt{8.98}}{3+\sqrt{4.8}}$

Can you give an estimate of the error?

(7) How would you suggest to define $f = f(x,y)$ to be "twice differentiable" at (x_0, y_0)?

Miscellaneous: Power series expansion, $f(g(x))$, $\dfrac{f(x)}{g(x)}$, ... etc.

Polar coordinate on the plane

Calculus on parametrized curves.

Applications of power series expansion.