(b) Suppose $f_x(0,0) = f_y(0,0) = f_{xy}(0,0) = f_{xx}(0,0) = f_{yy}(0,0) = 0$

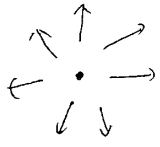
..... and f has a local minimum at $(0,0)$

show that $f_{xxx}(0,0) = f_{xxy}(0,0) = f_{xyy}(0,0) = f_{yyy}(0,0) = 0$ (15%).

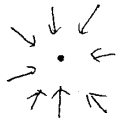
Hint : look at $f(x,0)$, $f(0,y)$

and then $f(x,x)$, $f(x,-x)$

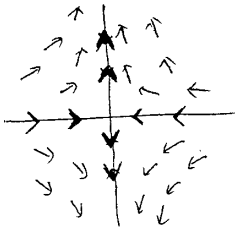
(7) Following are three local gradient diagrams near critical points. the arrows are pointing along the direction of $\nabla f(x,y)$



(A)

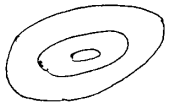


(B)

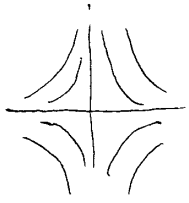


(C)

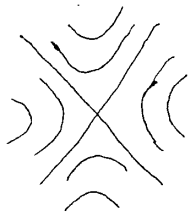
(a) For each of (A), (B), (C), what are possible diagrams of level curves of f ? (15%)



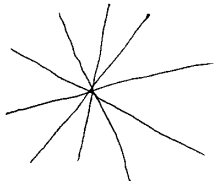
(D)



(E)



(F)



(G)

(b) For the diagram (A), what can you conclude about the sign of $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$ at the critical point ? (10%)

(Assume all partial derivatives are continuous)

(Be Careful!)