# MID TERM EXAM

## Summer 1999

Name: _____

1. (a) Find a Taylor series around $x = 0$ with radius of convergence $= 1$
   (b) Find a Taylor series that converges on $(0, 1]$ but diverges otherwise
2. (a) Write down the Taylor expansion up to the second order for $f(x, y)$ ($f$ has continuous derivative of all orders)
   (b) construct a second order polynomial in x,y such that $f(0,0) = 0, f_x(0,0) = 0, f_y(0,0) = 0$ and $(f_{xx}, f_{xy}, f_{yy}) = (1, 0, -1)$. And then find points $(x_1, y_1)$ and $(x_2, y_2)$ such that $f(x_1, y_1) > f(0,0), f(x_2, y_2) < f(0,0)$
3. $f(x) = \int_0^{\frac{1}{x}} \frac{\sin tx}{t} dt, x > 0$ , $\frac{d}{dx} f(x) = ?$
4. Let $f(x,y) = (x^2 + y^2)\sin(\frac{1}{\sqrt{x^2+y^2}})$,if $(x,y) \neq (0,0)$ and

   $f(x,y) = 0$,if $(x,y) = (0,0)$

   (a) Calculate $f_x(0,0)$ and $f_y(0,0)$
   (b) Explain why $f_x$ and $f_y$ not continuous at $(0,0)$
   (c) Show that $f$ is differentiable at $(0,0)$
5. Suppose $\int_{h_1(x)}^{h_2(x)} f(x,y,t)dt = 0$ is relation between x and y , actually defines y as a function of x. If we write y=$\phi(x)$ for this function. Assume all functions $h_1, h_2, \phi, f$ are continuously differentiable. Find $\phi'(x)$
6. Test for convergence or not
   (a) $\sum_{n=3}^{\infty} (\frac{n}{n+1})^n$
   (b) $\Pi_{n=1}^{\infty} (n^2)^{1/n}$
7. Let $b_1, b_2, ....., b_k$ be positive numbers. Find the maximum value of $\sum_{i=1}^{k} b_i x_i$ subject to the side condition $\sum_{i=1}^{k} x_i^2 = 1$