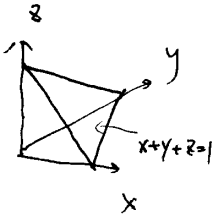
1 (15 pts) Write down the spherical coordinate system, (i.e. express (x, y, z) as functions of (r, θ, φ)) then compute the Jacobian $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$.

2 (15 pts)



Let Ω be the region bounded by the 3 coordinate planes and $x+y+z=1$ in the first Octant. Compute the volume of Ω by $\iiint dx\,dy\,dz$

(specify the upper and lower limits of integration with respect to this particular order. then compute the integral)

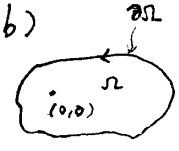
3 (15 pts)

Compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the upper half of the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$, counter clockwise oriented and $\vec{F}(x,y) = (-y, 3x)$

4 (15 pts) Give an explicit example of $\vec{F} = (F_1(x,y,z), F_2(x,y,z), F_3(x,y,z))$ such that $\int_{(x_0,y_0,z_0)}^{(x_1,y_1,z_1)} \vec{F} \cdot d\vec{r}$ is independent of the path joining (x_0, y_0, z_0) to (x_1, y_1, z_1). (~~examples~~ $\vec{F} \equiv (0,0,0)$ doesn't count)
(Give reason why your answer works)

5 (15 pts) Write down Green's Theorem on the plane (a) (tangential form) and prove it for a rectangular domain

(15 pts) (b)



~~Compute~~ Evaluate $\oint_{\partial\Omega} \left(\frac{x}{x^2+y^2} + x\right) dy - \frac{y}{x^2+y^2}\,dx$ using Green's Theorem.
Express your answer in terms of the area of Ω

6 (20 pts) (a) (~~15 pts~~) Express $\iint_{\substack{x^2+y^2+z^2=1 \\ z>0}} z\,d\sigma$ as a double integral (need not compute it)

(20 pts) (b) Find the answer by applying divergence theorem on the hemisphere $\{x^2+y^2+z^2<1,\ z>0\}$

[Hint: the boundary of the hemisphere has 2 portions, one of them is the surface given in (a). where the outer normal $\vec{n} = (x, y, z)$]