Brief answers to Quiz 1

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1. **(20 pts)** Give formal definition of $\lim_{n\to\infty} a_n = L$

   Ans.
   See page 552 of the textbook. If you did not get full credit, make sure you realize what was the mistake in your answer.

2. **(16 pts)** Is $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\tan t}\; dt$ convergent? Explain.

   Ans.
   $\because \lim_{t\to\pi/2-} \frac{\sqrt{\tan t}}{\frac{1}{\sqrt{\frac{\pi}{2}-t}}} = 1$. **(6 pts)** And, $\int_0^{\pi/2} \frac{1}{\sqrt{\frac{\pi}{2}-t}}\, dt = \int_0^{\pi/2} \frac{1}{\sqrt{u}}\, du$ converges. **(6 pts)**
   $\therefore$ By Limit Comparison Test, $\int_0^{\frac{\pi}{2}} \sqrt{\tan t}\, dt$ converges. **(4 pts)**

3. **(16 pts)** Is $\displaystyle\sum_{n=1}^{\infty} (1+\frac{1}{n})^n$ convergent? Explain.

   Ans.
   $\because (1+\frac{1}{n})^n \to e$ **(12 pts)**.
   $\therefore$ By Nth Term Test, $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$ divergent **(4 pts)**.

4. **(16 pts)** Is $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$ convergent? Explain.

   Ans.
   Let $f(x) = \frac{1}{x(\ln x)^2}$ **(3 pts)**. Then f is positive, continuous, and decreasing on $[3,\infty)$ **(3 pts)**.
   $\because \int_3^{\infty} \frac{1}{x(\ln x)^2}\, dx = \int_{\ln 3}^{\infty} \frac{1}{u^2}\, du$ converges. **(6 pts)**
   $\therefore$ By Integral Test, $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$ converges **(4 pts)**.

5. **(16 pts)** Is $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ convergent? Explain.

   $\because \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^3+1}}}{\frac{1}{n^{\frac{3}{2}}}} = 1$ **(6 pts)**. And, $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ converges **(6 pts)**.
   $\therefore$ By Limit Comparison Test, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges **(4 pts)**.

6. **(16 pts)** Is $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$ convergent? Explain.

Ans.
Let $a_n = \frac{2^n n! n!}{(2n)!}$.
$\because \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2(n+1)^2}{(2n+1)(2n+2)} = \frac{1}{2}$ **(12 pts)**.
$\therefore$ By Ratio Test, $\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$ converges **(4 pts)**.