

Solutions to selected problems in HW for Week 16

1. Section.16.5: Problem 56.

- (a) Fix $x \in [a, b]$, then y, z satisfy the equation of circle $y^2 + z^2 = f(x)^2$. Therefore we have the parametrization:

$$r(x, \theta) = (x, f(x) \cos \theta, f(x) \sin \theta), \quad 0 \leq \theta \leq 2\pi, \quad a \leq x \leq b.$$

- (b) First compute

$$\begin{aligned} r_x &= (1, f'(x) \cos \theta, f'(x) \sin \theta), \\ r_\theta &= (0, -f(x) \sin \theta, f(x) \cos \theta), \\ |r_x \times r_\theta| &= \sqrt{(f(x)f'(x))^2 + f(x)^2} = |f(x)|\sqrt{1 + (f'(x))^2}. \end{aligned}$$

Thus,

$$Area = \int_0^{2\pi} \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx d\theta = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx.$$