# Solutions to selected problems in HW for Week 10

1. Homework assignment: Problem 2.

   Short summary:

   - All nearby gradient vectors point towards the critical point: local maximum.
   - All nearby gradient vectors point away from the critical point: local minimum.
   - Some nearby gradient vectors point towards the critical point, some nearby gradient vectors point away from the critical point: saddle point.

2. Section 14.10: Problem 12.

$$\begin{cases} \frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}+\frac{\partial f}{\partial w}\frac{\partial w}{\partial x}=0 \\ \frac{\partial g}{\partial x}+\frac{\partial g}{\partial z}\frac{\partial z}{\partial x}+\frac{\partial g}{\partial w}\frac{\partial w}{\partial x}=0 \end{cases} \Rightarrow \frac{\partial z}{\partial x}=-\frac{f_x g_w - f_w g_x}{f_z g_w - f_w g_z}$$

$$\begin{cases} \frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial y}+\frac{\partial f}{\partial w}\frac{\partial w}{\partial y}=0 \\ \frac{\partial g}{\partial y}+\frac{\partial g}{\partial z}\frac{\partial z}{\partial y}+\frac{\partial g}{\partial w}\frac{\partial w}{\partial y}=0 \end{cases} \Rightarrow \frac{\partial w}{\partial y}=-\frac{f_z g_y - f_y g_z}{f_z g_w - f_w g_z}$$

3. Homework assignment: Problem 5.
   Note that

$$\left(\frac{\partial u}{\partial x}\right)_y=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\frac{\partial z}{\partial x}+\frac{\partial u}{\partial w}\frac{\partial w}{\partial x}.$$

   We've known from Problem.12 that

$$\frac{\partial z}{\partial x}=-\frac{f_x g_w - f_w g_x}{f_z g_w - f_w g_z}.$$

   For another unknown term, consider

$$\begin{cases} \frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}+\frac{\partial f}{\partial w}\frac{\partial w}{\partial x}=0 \\ \frac{\partial g}{\partial x}+\frac{\partial g}{\partial z}\frac{\partial z}{\partial x}+\frac{\partial g}{\partial w}\frac{\partial w}{\partial x}=0 \end{cases} \Rightarrow \frac{\partial w}{\partial x}=-\frac{f_x g_z - f_z g_x}{f_w g_z - f_z g_w}.$$

   Therefore,

$$\left(\frac{\partial u}{\partial x}\right)_y=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\left(-\frac{f_x g_w - f_w g_x}{f_z g_w - f_w g_z}\right)+\frac{\partial u}{\partial w}\left(-\frac{f_x g_z - f_z g_x}{f_w g_z - f_z g_w}\right).$$

4. Homework assignment: Problem 6.

$$\begin{aligned}\frac{d}{dx}f(x,y(x),z(x))\Big|_{x=1/\sqrt{2}} &= 2x+2y\left(-\frac{x}{y}\right)+2z\left(-1+\frac{x}{y}\right)\Big|_{(1/\sqrt{2},1/\sqrt{2},1-\sqrt{2})} \\ &= 2z\left(-1+\frac{x}{y}\right)\Big|_{(1/\sqrt{2},1/\sqrt{2},1-\sqrt{2})}=0.\end{aligned}$$