Solutions to selected problems in HW for Week 08

1. Section 14.5: Problem 36.
   (a) The maximal value of $D_v f(P)$ is $|\nabla f(P)|$. Therefore, $|\nabla f(P)| = 2\sqrt{3}$. And thus, $\nabla f(P) = |\nabla f(P)| \cdot \frac{v}{|v|} = 2\sqrt{3} \cdot (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}) = (2, 2, -2)$.
   (b) $D_u f(P) = \nabla f(P) \cdot (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0) = 2\sqrt{2}$.

2. Section 14.5: Problem 40.
   Write $x = (x_1, x_2, x_3)$.

$$
\begin{aligned}
&(\nabla(f \pm g))_i = \partial_{x_i}(f \pm g) = \partial_{x_i} f \pm \partial_{x_i} g = (\nabla f)_i \pm (\nabla g)_i \\
&(\nabla(kf))_i = \partial_{x_i}(kf) = k\partial_{x_i} f = k(\nabla f)_i \\
&(\nabla(fg))_i = \partial_{x_i}(fg) = (\partial_{x_i} f)g + f(\partial_{x_i} g) = (\nabla f)_i g + f(\nabla g)_i \\
&(\nabla(f/g))_i = \partial_{x_i}(f/g) = ((\partial_{x_i} f)g - (\partial_{x_i} g)f)/g^2 = ((\nabla f)_i g - (\nabla g)_i f)/g^2
\end{aligned}
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3. Homework assignment: Problem 2.
   (a) Yes. $0 \le |f(r\cos\theta, r\sin\theta)| = |r\cos^2\theta\sin\theta| \le r \to 0 = f(0,0)$ as $r \to 0$.
   (b) Yes.

$$
\begin{aligned}
\lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} &= \lim_{h\to 0} \frac{0-0}{h} = 0 \Rightarrow f_x(0,0) = 0, \\
\lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} &= \lim_{h\to 0} \frac{0-0}{h} = 0 \Rightarrow f_y(0,0) = 0.
\end{aligned}
$$

   (c)

$$
\lim_{s\to 0} \frac{f(s\cos\theta, s\sin\theta)}{s} = \lim_{s\to 0} \cos^2\theta\sin\theta = \cos^2\theta\sin\theta.
$$

   (d) No, for the reason that $D_u f(0,0) \ne \nabla f(0,0) \cdot u = 0$ if $u = (\cos\theta, \sin\theta)$ with $\cos^2\theta\sin\theta \ne 0$.