Solutions to selected problems in HW for Week 02

1. Section 10.4: Problem 61.
   Take $r \in (1, p)$. Consider
   $$\lim_{n\to\infty} \frac{\frac{(\ln n)^q}{n^p}}{\frac{1}{n^r}} = \lim_{n\to\infty} \frac{(\ln n)^q}{n^{p-r}} = 0.$$
   And,
   $$\sum_n \frac{1}{n^r} \text{ converges.}$$
   By Limit Comparison Test, the series converges.

2. Section 10.4: Problem 62.
   $\boxed{0 < p < 1}$
   Take $r \in (p, 1)$. Consider
   $$\lim_{n\to\infty} \frac{\frac{(\ln n)^q}{n^p}}{\frac{1}{n^r}} = \lim_{n\to\infty} n^{r-p}(\ln n)^q = \infty.$$
   And,
   $$\sum_n \frac{1}{n^r} \text{ diverges.}$$
   By Limit Comparison Test, the series diverges.

   $\boxed{p = 1}$ Use Integral Test with the function $f(x) = \frac{(\ln x)^q}{x}$.
   $q \neq -1$:
   $$\int_2^\infty \frac{(\ln x)^q}{x} = \int_{\ln 2}^\infty u^q\,du = \begin{cases} = \infty & \text{if } q > -1, \\ < \infty & \text{if } q < -1. \end{cases}$$
   $q = -1$:
   $$\int_2^\infty \frac{(\ln x)^{-1}}{x}\,dx = \int_2^\infty \frac{1}{u}\,du = \ln u \Big|_2^\infty = \infty.$$