

Solution of Quiz 2

Oct 20, 2015

1. Evaluate
- $\frac{d^4}{dx^4}(x^4 \cos(x-1))|_{x=1}$
- .

Ans:

$$\begin{aligned}
& \left. \frac{d^4}{dx^4}(x^4 \cos(x-1)) \right|_{x=1} \\
&= \left[x^4 \frac{d^4}{dx^4}(\cos(x-1)) + 4 \frac{d}{dx}(x^4) \frac{d^3}{dx^3}(\cos(x-1)) + 6 \frac{d^2}{dx^2}(x^4) \frac{d^2}{dx^2}(\cos(x-1)) \right. \\
&+ \left. 4 \frac{d^3}{dx^3}(x^4) \frac{d}{dx}(\cos(x-1)) + \left(\frac{d^4}{dx^4}(x^4) \right) \cos(x-1) \right] \Big|_{x=1} \quad \textbf{(9 points)} \\
&= [x^4 \cos(x-1) + 4(4x^3) \sin(x-1) + 6(12x^2)(-\cos(x-1)) \\
&+ 4(24x)(-\sin(x-1)) + 24 \cos(x-1)] \Big|_{x=1} \quad \textbf{(9 points)} \\
&= 1 + 0 - 72 + 0 + 24 = -47. \quad \textbf{(2 points)}
\end{aligned}$$

2. Find the derivative of
- $y = \tan(e^{\sqrt{x^2+1}})$
- . Need not simplify your final expression.

Ans:

$$\frac{dy}{dx} = \sec^2(e^{\sqrt{x^2+1}}) e^{\sqrt{x^2+1}} \frac{2x}{2\sqrt{x^2+1}}.$$

Each term: **(6 points)(6 points)(8 points)**.

3. Find all roots of
- $y'' = 0$
- if
- $y = (1 + \frac{1}{x})^3$
- .

Ans:

$$\begin{aligned}
y' &= -3(1 + \frac{1}{x})^2 x^{-2} \quad \textbf{(9 points)} \\
y'' &= 6 \left[(1 + \frac{1}{x}) \frac{1}{x^4} + (1 + \frac{1}{x})^2 \frac{1}{x^3} \right] = 6(1 + \frac{1}{x}) \frac{1}{x^3} (1 + \frac{2}{x}) = 0 \quad \textbf{(9 points)} \\
x &= -1, -2. \quad \textbf{(2 points)}
\end{aligned}$$

4. Use implicit differentiation (and not other methods) to find
- dy/dx
- and
- d^2y/dx^2
- at
- $(1,1)$
- where
- $y(x)$
- is implicitly given by
- $x^4 + y^4 = 2$
- .

Ans:

$$\frac{dy}{dx} : \quad x^4 + y^4 = 2 \Rightarrow 4x^3 + 4y^3 \frac{dy}{dx} = 0 \quad \textbf{(5 points)}$$

$$(x, y) = (1, 1) \Rightarrow 4 + 4 \frac{dy}{dx} \Big|_{(1,1)} = 0$$

$$\Rightarrow \frac{dy}{dx} \Big|_{(1,1)} = -1 \quad \textbf{(5 points)}$$

$$\frac{d^2y}{dx^2} : \quad 4x^3 + 4y^3 \frac{dy}{dx} = 0 \Rightarrow 12x^2 + 12y^2 \left(\frac{dy}{dx} \right)^2 + 4y^3 \frac{d^2y}{dx^2} = 0 \quad \textbf{(5 points)}$$

$$(x, y) = (1, 1) \Rightarrow 12 + 12 \cdot (-1)^2 + 4 \frac{d^2y}{dx^2} \Big|_{(1,1)} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} \Big|_{(1,1)} = -6. \quad \textbf{(5 points)}$$

5. True or False? (prove it if true, correct it if false). If f , g and h are differentiable functions on R and $f(g(x)) = h(x)$. Let $\frac{d}{dx}f(x) = f_1(x)$, $\frac{d}{dx}g(x) = g_1(x)$, $\frac{d}{dx}h(x) = h_1(x)$. Then $f_1(x) \cdot g_1(x) = h_1(x)$.

Ans:

i It's false. **(5 points)**

ii It should be $f_1(g(x)) \cdot g_1(x) = h_1(x)$. **(15 points)**