Brief answers to Quiz 1

Oct 06, 2015

1. Give formal definition of $\lim_{x\to c} f(x) \neq L$ (just the definition, need not find ϵ or δ, etc.).

   Ans:

   See Remark 2 in 'Study Guide for Chap 02 (New!)'.

2. Find $\lim_{\theta\to 0} \dfrac{\sin(1-\cos\theta)}{\tan^2\theta}$.

   Ans: 1/2. (See the hint in problem 3 of Homework 02)

3. State the Intermediate Value Theorem (Need not prove). Use it to show that $x^x = 2$ has a root.

   Ans:

   For the second part, apply Intermediate Value Theorem to the function $g(x) = x^x - 2$ on the interval $[1, 2]$.

4. Give formal definition of '$y = f(x)$ is continuous at $x = c$' in terms of ϵ and δ. Then use the $\epsilon - \delta$ argument to show that if both $f(x)$ and $g(x)$ are continuous at $x = c$, then so is $f(x) + g(x)$.

   Ans:

   The first part:

   For any $\varepsilon > 0$, there exists $\delta > 0$ such that,

   $$|x - c| < \delta \Longrightarrow |f(x) - f(c)| < \varepsilon$$

   The second part:

   The proof is similar (and almost identical) to Example 6 of section 2.3. Just replace L and M by $f(c)$ and $g(c)$, respectively. See page 81 for details.

5. Give formal definitions of the following limits (Just the definition, need not find δ).

   $$(a) \lim_{x\to c^-} f(x) = L \qquad\qquad (b) \lim_{x\to -\infty} f(x) = \infty$$

   Ans:

   (a) For any $\varepsilon > 0$, there exists $\delta > 0$ such that,

   $$c - \delta < x < c \Longrightarrow |f(x) - L| < \varepsilon$$

   (b) For any $B > 0$, there exists $N < 0$ such that

   $$x < N \Longrightarrow f(x) > B$$