Brief answer to selected problems in HW09

1. Section 4.5:

   problem 80: $a = -2$, $b = -\frac{8}{3}$.

   problem 84(c): Answer = 1.

   problem 86: Solve for $\frac{d}{dx} x^{\frac{1}{x^n}} = 0$, one gets $x^{\frac{1}{x^n}} \frac{1 - n \ln x}{x^{n+1}} = 0$. So the only critical point is $x = e^{\frac{1}{n}}$. Next show that $x = e^{\frac{1}{n}}$ is indeed a maximum by first derivative test. Answer: Maximum $= e^{\frac{1}{ne}}$.

2. Section 4.6:

   problem 12: maximize $V(y) = \frac{1}{3}\pi(9 - y^2)(3 + y)$ on $0 \le y \le 3$. It is easier to calculate than using $V(x)$.

   problem 48: Similar to Example 4. Minimize $t = \dfrac{\sqrt{a^2 + x^2}}{c} + \dfrac{\sqrt{b^2 + (d - x)^2}}{c}$ with respect to x,

3. Section 4.7:

   Problem 18: Solve, for example, $f(x) = \tan(\frac{x}{4}) - 1 = 0$.

   Problem 29: The solution: $x_* = 1$. Newton's method: $x_{n+1} = x_n - \dfrac{x_n - 1}{40}$. Therefore the error between nth and $(n+1)$th iteration is given by $x_{n+1} - x_* = \dfrac{39}{40}(x_n - x_*)$. It takes about $\dfrac{3}{\log_{10}(\frac{40}{39})} \approx 118$ iterations.

   Problem 30: Combine $r\theta = 3$ and $r \sin(\frac{\theta}{2}) = 1$ to get $f(r) = r \sin(\frac{3}{2r}) - 1 = 0$ and solve for r.