Brief answer to selected problems in HW07

1. Section 3.11: Problem 64.

   (a)$b_0 = f(a)$, $b_1 = f'(a)$, and $b_2 = \frac{f''(a)}{2}$

   (b)Quadratic approximation of $\frac{1}{1-x}$ at $x = 0$ is $1 + x + x^2$

   (d)Quadratic approximation of $\frac{1}{x}$ at $x = 1$ is $1 - (x-1) + (x-1)^2$

   (e)Quadratic approximation of $\sqrt{1+x}$ at $x = 0$ is $1 + \frac{1}{2}x - \frac{1}{8}x^2$

   Quadratic approximations are better than linear approximations near the base point x_0 in general.

2. Error estimate for the linear approximation of section 3.11, problem 17 (b):

   For $\sqrt[3]{1.009}$, let

   $$f(x) = (1+x)^{\frac{1}{3}} \Longrightarrow f''(x) = -\frac{2}{9}(1+x)^{-\frac{5}{3}}$$

   Then $\sqrt[3]{1.009} = f(x),\ \ x = 0.009$. Hence we can use linear approximation near $x_0 = 0$. By the error formula, we can get error bound by following process.

   $$\begin{aligned} |f(0.009) - L(0.009;0)| &\leq \frac{1}{2}(\max_{0\leq c\leq 0.009} |f''(c)|)(0.009-0)^2 \\ &= \frac{1}{2} \times \frac{2}{9} \times 0.009^2 = 9 \times 10^{-6} \end{aligned}$$

3. Chap 3, additional and advanced problem.

   Problem 21:

   By definition, find the limit

   $$\lim_{x\to x_0} \frac{f(x)g(x) - f(x_0)g(x_0)}{x - x_0}.$$

   Use the fact that $f(x_0) = 0$ to replace $f(x_0)g(x_0)$ by $f(x_0)g(x)$. Then take the limit and conclude that answer $= f'(x_0)g(x_0)$.

   Problem 23:

   (1) Find $h'(x)$ for $x \neq 0$ from straight forward calculation.

   (2) Find $h'(0)$ from definition $h'(0) = \lim_{x\to 0} \frac{h(x)-h(0)}{x}$.

   (3) Check whether or not $\lim_{x\to 0} h'(x) = h'(0)$.

   The procedure for $k(x) = xh(x)$ is similar.