Brief answer to selected problems in HW02

1. No. For example. $\lim_{x\to 1} f(x) = 1$. Yet, for any $\delta > 0$, one can take $\epsilon_0 = \delta/4$ and $x_0 = 1 + \delta/4$ satisfying the definition.

2. Section 2.4: Problem 26 (answer = 2), 34 (answer = 1), 42 (answer = 1).

3. Section 2.5:

   Problem 64: If f is discontinuous at $g(0)$, $f(g(x))$ may still be discontinuous at $x = 0$. No contradiction.

4. Chapter 2: Problem 25 (answer = 0), 26 (answer = 0).

5. Section 2.5:

   Problem 67: The function $g(x) = f(x) - x$ satisfies $g(0) \cdot g(1) \leq 0$. If the equality holds, then c is found. Otherwise, apply the Intermediate Value Theorem to $g(x)$ on $[0, 1]$.

   Problem 77: The function $f(x) = \cos(x) - x$ satisfies $f(0) \cdot f(1) < 0$.

6. Section 2.6:

   Problem 92: If $B > 0$, take $\delta = 1/\sqrt{B}$ will do. If $B \leq 0$, take δ to be any positive real number (for example $\delta = 1$) will do.

   Problem 93:

   (a): change '$x_0 < x < x_0 + \delta$' to '$x_0 - \delta < x < x_0$'.

   (b): change '$f(x) > B$ to $f(x) < B$ (or $f(x) < -B$).

   (c): apply both changes in (a) and (b).

   Problem 100:

   $y = x + 1 + \frac{2}{x-1}$. Vertical asymptote: $x = 1$. Oblique asymptote: $y = x + 1$.