# Solutions in Quiz04

1. $\lim_{x\to 0}(\frac{1}{x}-\frac{1}{\sin x}) = \lim_{x\to 0}\frac{\sin x - x}{x\sin x} = \lim_{x\to 0}\frac{\cos x - 1}{\sin x + x\cos x} = \lim_{x\to 0}\frac{-\sin x}{2\cos x - x\sin x} = 0.$

2. $\lim_{x\to\infty}\frac{e^{x^2}}{xe^x} = \lim_{x\to\infty}\frac{e^{x^2-x}}{x} = \lim_{x\to\infty}\frac{e^{x^2-x}(2x-1)}{1} = \infty.$

3. Let $D^2 = (x-2)^2 + (y-0)^2 = (x-2)^2 + x$ on $y = \sqrt{x}$ and $x \geq 0$.
   Then $\frac{dD^2}{dx} = 2x - 3$, hence $x = \frac{3}{2}$ is a critical point.
   Moreover $\frac{d^2D^2}{dx^2} = 2 > 0$ for all $x \geq 0$.
   Hence $(\frac{3}{2}, \sqrt{\frac{3}{2}})$ is the point on $y = \sqrt{x}$ which is closest to (2,0).

4. Let $f(x) = x^3 - 2$, then given $x_0 = 1$, compute $x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2}$.

5. $\int x^2 + 2^x dx = \frac{x^3}{3} + \frac{1}{\ln 2}2^x + c$ for some constant $c \in R$.