Solutions in Quiz03

1. If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is one point c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. Absolute maximum is $f(-3) = 6\sqrt[3]{9}$
   Absolute minimum is $f(3) = f(0) = 0$.

3. It's true. Let $f(x) = \cos x$, then $f(x)$ is continuous and differentiable on R. For all values of x, by Mean Value Theorem,

$$\frac{\cos x - \cos 0}{x - 0} = \sin c$$

   for some $c \in (0, x)$.

   Hence

$$|\cos x - 1| \leq |x|$$

   for all values of x.

4. Let $f(x) = e^x - 1 - x$, then $f(0) = 0$ and $f'(x) = e^x - 1 \geq 0$ on $x \geq 0$.

   Hence $f(x) \geq f(0) = 0$ on $x \geq 0$. That is $e^x - 1 - x \geq 0$ on $x \geq 0$.

   Let $g(x) = e^x - 1 - x - \frac{x^2}{2}$, then $g(0) = 0$ and $g'(x) = e^x - 1 - x \geq 0$ on $x \geq 0$.

   Hence $g(x) \geq g(0) = 0$ on $x \geq 0$. That is $e^x \geq 1 + x + \frac{x^2}{2}$ on $x \geq 0$.

5. $f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$ and $f''(x) = 12x^2 - 24x = 12x(x - 2)$. We have

$$\begin{cases} \text{Critical points: x=0, 3} \\ \text{Points of inflection: x=0, 2} \end{cases}$$

   and $f(x)$

$$\begin{cases} \text{Decreasing and concave up on } (-\infty, 0] \\ \text{Decreasing and concave down on } [0, 2] \\ \text{Decreasing and concave up on } [2, 3] \\ \text{Increasing and concave up on } [3, \infty) \end{cases}$$