Brief Answers to Quiz 2

Show all details.

1. Evaluate $\dfrac{d^4}{dx^4}\big(x^4\cos(x-1)\big)|_{x=1}$.

   Ans:

$$\begin{aligned}\frac{d^4}{dx^4}(x^4\cos(x-1))|_{x=1} &= [24\cos(x-1) - 96x\sin(x-1) - 72x^2\cos(x-1) \\ &+ 16x^3\sin(x-1) + x^4\cos(x-1)]|_{x=1} \\ &= -47\end{aligned}$$

2. Find the derivative of $y = \tan(\exp(\sqrt{x^2+1}))$. Need not simplify your final expression.

   Ans:

$$\frac{dy}{dx} = \sec^2(\exp(\sqrt{x^2+1})) \times \exp(\sqrt{x^2+1}) \times \frac{x}{\sqrt{x^2+1}}$$

3. Suppose we know that $\frac{d}{dx}x^n = nx^{n-1}$ for all integers n. Show that, based on this fact, we can derive the same for $n = q/p$ where p, q are integers and $p \neq 0$.

   Ans:

   Let

$$y = x^{\frac{q}{p}} \Longrightarrow y^p = x^q$$

   Implicit differentiation gives us

$$py^{p-1}\frac{dy}{dx} = qx^{q-1}$$

   hence

$$\frac{d}{dx}x^{\frac{q}{p}} = \frac{dy}{dx} = \frac{q}{p}\frac{x^{q-1}}{y^{p-1}} = \frac{q}{p}\frac{x^{q-1}}{(x^{\frac{q}{p}})^{p-1}} = \frac{q}{p}x^{q-1-\frac{q}{p}(p-1)} = \frac{q}{p}x^{\frac{q}{p}-1}$$

   that is

4. Use implicit differentiation (and not other methods) to find dy/dx and d^2y/dx^2 at $(1,1)$ where $y(x)$ is implicitly given by $x^4 + y^4 = 2$.

   Ans: Using Implicit differentiation, we have

$$4x^3 + 4y^3\frac{dy}{dx} = 0$$

substitute $(x, y) = (1, 1)$ can get

$$4 + 4\frac{dy}{dx}|_{(1,1)} = 0 \Longrightarrow \frac{dy}{dx}|_{(1,1)} = -1$$

using Implicit differentiation again to get

$$12x^2 + 12y^2(\frac{dy}{dx})^2 + 4y^3\frac{d^2y}{dx^2} = 0$$

substitute $(x, y) = (1, 1)$ can get

$$12 + 12(-1)^2 + 4\frac{d^2y}{dx^2}|_{(1,1)} = 0 \Longrightarrow \frac{d^2y}{dx^2}|_{(1,1)} = -6$$

5. True or False? (prove it if true, correct it if false).

   If f, g and h are differentiable functions on R and $f(g(x)) = h(x)$. Let $\frac{d}{dx}f(x) = f_1(x)$, $\frac{d}{dx}g(x) = g_1(x)$, $\frac{d}{dx}h(x) = h_1(x)$. Then $f_1(x) \cdot g_1(x) = h_1(x)$.

   Ans:

   False. Correct version:

$$f_1(g(x)) \cdot g_1(x) = h_1(x)$$