Brief answer to selected problems in Homework 11

1. Section 5.6:

   problem 112: use the change of variable $u = 1 - x$.

   problem 117: use the change of variable $u = x - c$.

2. Chapter 5:

   problem 4: $1 = \frac{d}{dx}x = \frac{d}{dy}\int_0^y \frac{1}{\sqrt{1+4t^2}}dt \times \frac{dy}{dx}$, implies $\frac{dy}{dx} = \sqrt{1+4t^2}$. Hence

   $$\frac{d^2y}{dx^2} = 4y$$

   problem 20: $\lim\limits_{x\to\infty} \frac{1}{x}\int_0^x \tan^{-1} t dt = \frac{\pi}{2}$

   problem 22: $\lim\limits_{n\to\infty} \frac{1}{n}(e^{\frac{1}{n}} + e^{\frac{2}{n}} + \cdots + e^{\frac{n-1}{n}} + e^{\frac{n}{n}}) = \int_0^1 e^x dx$

   problem 26:

   a. $\lim\limits_{n\to\infty} \frac{1}{n^2}(2+4+6+\cdots+2n) = \lim\limits_{n\to\infty} \frac{1}{n}(2\frac{1}{n} + 2\frac{2}{n} + \cdots + 2\frac{n}{n}) = \int_0^1 2xdx$

   b. $\lim\limits_{n\to\infty} \frac{1}{n^{16}}(1^{15} + 2^{15} + \cdots + n^{15}) = \lim\limits_{n\to\infty} \frac{1}{n}((\frac{1}{n})^{15} + (\frac{2}{n})^{15} + \cdots + (\frac{n}{n})^{15}) = \int_0^1 x^{15}dx$

   c. $\lim\limits_{n\to\infty} \frac{1}{n}(\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \cdots + \sin\frac{n\pi}{n}) = \int_0^1 \sin \pi x dx$

   By b, we can derive the following

   d. $\lim\limits_{n\to\infty} \frac{1}{n^{17}}(1^{15} + 2^{15} + \cdots + n^{15}) = \lim\limits_{n\to\infty} [\frac{1}{n}((\frac{1}{n})^{15} + (\frac{2}{n})^{15} + \cdots + (\frac{n}{n})^{15})] \times \frac{1}{n} = 0$

   e. $\lim\limits_{n\to\infty} \frac{1}{n^{15}}(1^{15} + 2^{15} + \cdots + n^{15}) = \lim\limits_{n\to\infty} [\frac{1}{n}((\frac{1}{n})^{15} + (\frac{2}{n})^{15} + \cdots + (\frac{n}{n})^{15})] \times n = \infty$