Brief answer to selected problems in Homework 09

1. Section 4.7:

   Problem 18: Solve, for example, $f(x) = \tan(\frac{x}{4}) - 1 = 0$.

   Problem 29: The solution: $x_* = 1$. Newton's method: $x_{n+1} = x_n - \dfrac{x_n - 1}{40}$. Therefore the error between nth and $(n+1)$th iteration is given by $x_{n+1} - x_* = \dfrac{39}{40}(x_n - x_*)$. It takes about $\dfrac{3}{\log_{10}(\frac{40}{39})} \approx 118$ iterations.

   Problem 30: Combine $r\theta = 3$ and $r\sin(\frac{\theta}{2}) = 1$ to get $f(r) = r\sin(\frac{3}{2r}) - 1 = 0$ and solve for r.

2. Section 4.8:

   Problem 84: (b) and (c) are right.

   Problem 128: Yes. Since $F - G$ is a constant and $F(y_0) - G(y_0) = 0$.

3. Section 5.3:

   Problem 87: We take for granted from the problem that a continuous function on a closed interval $[a, b]$ must be uniformly continuous (ie. Assume this statement is correct. This is an advanced calculus Theorem).

   Therefore given $\epsilon > 0$, one can find $\delta > 0$ such that $|x_1 - x_2| < \delta$ implies $|f(x_1) - f(x_2)| < \epsilon$. It is not difficult to see that for this δ, $\|P\| < \delta$ implies $U - L < \epsilon(\sum_k \Delta x_k) = \epsilon(b - a)$.