Brief answer to selected problems in HW03

1. Section 3.2: Problem 48.

    a. $\{x| -3 \le x \le 3, and\ x \ne 2, \ne -2\}$

    b. $\{2, -2\}$

    c. $\phi(\text{empty set})$

2. Section 3.2: Problem 54.

    Assume tangent line intersect with the curve at (x_0, y_0). Then solve the equations for $x_0 \ge 0$

    $$\begin{cases} y_0 = \frac{1}{2\sqrt{x_0}}(x_0 + 1) \\ y_0 = \sqrt{x_0} \end{cases}$$

3. Section 3.2: Problem 57.

    Observing following limits for $\lim\limits_{t \to 0} \frac{g(t)}{h(t)}$

    $$(1)\begin{cases} g(t) = t^2 \\ h(t) = t \end{cases} \quad (2)\begin{cases} g(t) = t \\ h(t) = 2t \end{cases} \quad (3)\begin{cases} g(t) = t \\ h(t) = t^2 \end{cases}$$

4. Section 3.3: Problem 70.

    $$\begin{aligned} \text{continuity:} &\quad -a + b = b - 3 \\ \text{differentiable:} &\quad a = -2b \end{aligned}$$

5.

    $$\frac{d^n}{dx^n}(f(x)g(x)) = \sum_{k=0}^{n} C_k^n (\tfrac{d^k}{dx^k}f)(\tfrac{d^{n-k}}{dx^{n-k}}g)$$

6. Section 3.5: Problem 58.

    For continuity, we can find $b = 1$, and $g'(x)$ is not differentiable at $x = 0$ since

    $$\lim_{x \to 0^+} g'(x) \ne \lim_{x \to 0^-} g'(x)$$