Solutions in Quiz01

1. $\exists \epsilon > 0$, such that $\forall \delta > 0$, $\exists x_0 \in (c-\delta, c) \cup (c, c+\delta)$ and $|f(x_0) - L| \geq \epsilon$

2. $\lim\limits_{\theta \to 0} \frac{sin(sin\theta)}{tan2\theta} = \lim\limits_{\theta \to 0} \frac{sin(sin\theta)}{sin2\theta} \cdot cos2\theta = \lim\limits_{\theta \to 0} \frac{sin(sin\theta)}{sin\theta} \cdot \frac{cos2\theta}{2cos\theta} = \frac{1}{2}$

3. (i) Intermediate Value Theorem:
   If f is a continuous function on a closed interval $[a, b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.

   (ii) Let $g(x) = x - 1 - cosx$, then g is continuous on R.
   Since
   $$\begin{aligned} g(1) &= -cos1 < 0 \\ g(2) &= 1 - cos2 > 0 \end{aligned}$$
   By Intermediate Value Theorem, there exist $x_0 \in [1, 2]$ such that $g(x_0) = 0$, i.e. $x_0 - 1 = cosx_0$. Hence $c = 1$.

4. Since $f(x)$ and $g(x)$ are continuous at $x = c$, we have
   $\forall \epsilon > 0$, $\exists \delta_1 > 0, \delta_2 > 0$, such that for all x,
   $$\begin{cases} |x-c| < \delta_1 \Longrightarrow |f(x) - f(c)| < \frac{\epsilon}{5} \\ |x-c| < \delta_2 \Longrightarrow |g(x) - g(c)| < \frac{\epsilon}{5} \end{cases}$$
   Taking $\delta = min(\delta_1, \delta_2) > 0$, then for all x, $|x - c| < \delta$ implies
   $$|[2f(x) - 3g(x)] - [2f(c) - 3g(c)]| \leq 2|f(x) - f(c)| + 3|g(x) - g(c)| < \epsilon$$
   Hence $2f(x) - 3g(x)$ is also continuous at $x = c$.

5. (a)$\forall \epsilon > 0$, $\exists \delta > 0$ such that, for all x,
   $$c - \delta < x < c \Longrightarrow |f(x) - L| < \epsilon$$
   (b)$\forall B > 0$, $\exists N < 0$ such that, for all x,
   $$x < N \Longrightarrow f(x) > B$$