HW 2. 1. NO! Let $f(x)=x$, it's clear that $\lim_{x\to1} f(x)=1$

But for any $\delta>0$, $\exists\ \epsilon_0=\frac{\delta}{3}>0$ and $x_0=1+\frac{\delta}{2} \Rightarrow x_0\in(1-\delta,1)\cup(1,1+\delta)$

such that $|f(x_0)-1|=|x_0-1|=\frac{\delta}{2}>\epsilon_0$

Section 2.4 26. $\lim_{t\to0}\frac{2t}{\tan t}=\lim_{t\to0} 2\cdot\frac{t}{\sin t}\cos t=2$

34. $\lim_{h\to0}\frac{\sin(\sin h)}{\sin h}=\lim_{\sin h\to0}\frac{\sin(\sin h)}{\sin h}=1$

42. $\lim_{\theta\to0}\frac{\theta\cot4\theta}{\sin^2\theta\cot^2 2\theta}=\lim_{\theta\to0}\frac{\theta}{\sin\theta}\frac{1}{\sin\theta}\cdot\frac{\cos4\theta}{\sin4\theta}\cdot\frac{\sin^2 2\theta}{\cos^2 2\theta}$

$=\lim_{\theta\to0}\frac{\theta}{\sin\theta}\frac{\left(\frac{\sin2\theta}{2\theta}\right)^2}{\frac{\sin\theta}{\theta}\cdot\frac{\sin4\theta}{4\theta}}\cdot\frac{\cos4\theta}{\cos^2 2\theta}=1\cdot\frac{1^2}{1\cdot1}\cdot\frac{1}{1}=1$

Section 2.5 64. Let $f(x)=\begin{cases} x+1, & x>3 \\ x, & x\le3\end{cases}$, $g(x)=x+3$, $\forall x\in\mathbb{R}$

then f and g are continuous at $x=0$

but $f(g(x))=\begin{cases} x+4, & x>0 \\ x+3, & x\le0\end{cases}$ is discontinuous at $x=0$

67. Let $g(x)=f(x)-x$

$\because$ $f(x)$ and x are continuous on $[0,1]$, $\therefore$ g is continuous on $[0,1]$

$\because$ $g(0)=f(0)-0\ge0$, $g(1)=f(1)-1\le0$

$\Rightarrow g(1)\le0\le g(0)$

By Intermediate Value Theorem, $\exists\ c\in[0,1]$ s.t. $g(c)=0$

$\Rightarrow f(c)=c$

[redacted]

77. Let $f(x) = \cos x - x$, $\forall x \in \mathbb{R} \Rightarrow f$ is continuous on $\mathbb{R}$

$\because f(0) = \cos 0 - 0 = 1 > 0$ and $f(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \frac{\pi}{2} = -\frac{\pi}{2} < 0$

By Intermediate Value Theorem, $\exists\, c \in [0, \frac{\pi}{2}]$ s.t. $f(c) = 0$

$\Rightarrow \cos c = c$

$\Rightarrow \cos x = x$ has a solution

Section 2.6 92. $\forall N > 0$, $\exists \delta = \frac{1}{2\sqrt{N}}$ s.t. for all x, $0 < |x - (-5)| < \delta$

then $f(x) = \frac{1}{(x+5)^2} > \frac{1}{\delta^2} = 4N > N$

93. (a) $\lim_{x \to x_0^-} f(x) = \infty$

If, for every positive real number B, there exists a corresponding number $\delta > 0$ such that for all x

$x_0 - \delta < x < x_0 \Rightarrow f(x) > B$

(b) $\lim_{x \to x_0^+} f(x) = -\infty$

If, for every positive real number B, there exists a corresponding number $\delta > 0$ such that for all x

$x_0 < x < x_0 + \delta \Rightarrow f(x) < -B$

(c) $\lim_{x \to x_0^-} f(x) = -\infty$

If, for every positive real number B, there exists a corresponding number $\delta > 0$ such that for all x

$x_0 - \delta < x < x_0 \Rightarrow f(x) < -B$

[redacted]

5. (a) $\lim_{x \to 0^+} \frac{1}{x} = \infty$

For any $B > 0$, pick $\delta = \frac{1}{B} > 0$, then for all x,

$$0 < x < 0 + \delta = \frac{1}{B} \Rightarrow \frac{1}{x} > B$$

$$\Rightarrow \lim_{x \to 0^+} \frac{1}{x} = \infty$$

(b) $\lim_{x \to \infty} -x^2 = -\infty$

For any $B > 0$, pick $N = \sqrt{B} > 0$, then for all x,

$$x > N = \sqrt{B} \Rightarrow -x^2 < -B$$

$$\Rightarrow \lim_{x \to \infty} -x^2 = -\infty$$

[redacted]