Section 2.2 77. let

$$g(x) = \begin{cases} x^2, & x > 1 \\ x^4, & -1 \le x \le 1 \\ x^2, & x < -1 \end{cases}$$

$$h(x) = \begin{cases} x^4, & x > 1 \\ x^2, & -1 \le x \le 1 \\ x^4, & x < -1 \end{cases}$$

then $g(x) \le f(x) \le h(x)$, $\forall x \in \mathbb{R}$

$\because \lim_{x\to 1} g(x) = 1$ and $\lim_{x \to 1} h(x) = 1$

$\therefore$ By Sandwich Theorem, $\lim_{x\to 1} f(x) = 1$

$\because \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} h(x) = 0$

$\therefore$ By Sandwich Theorem, $\lim_{x\to 0} f(x) = 0$

$\because \lim_{x\to -1} g(x) = 1$ and $\lim_{x\to -1} h(x) = 1$

$\therefore$ By Sandwich Theorem, $\lim_{x\to -1} f(x) = 1$

87. $$\lim_{x\to 0} \frac{\sqrt[3]{1+x} - 1}{x} = \lim_{x\to 0} \frac{\sqrt[3]{1+x}-1}{x} \cdot \frac{(\sqrt[3]{1+x})^2 + \sqrt[3]{1+x} + 1}{(\sqrt[3]{1+x})^2 + \sqrt[3]{1+x} + 1} = \lim_{x\to 0} \frac{(\sqrt[3]{1+x})^3 - 1}{x[(\sqrt[3]{1+x})^2 + \sqrt[3]{1+x} + 1]}$$

$$= \lim_{x\to 0} \frac{1}{(\sqrt[3]{1+x})^2 + \sqrt[3]{1+x} + 1} = \frac{1}{3}$$

Section 2.3 49. $\forall \epsilon > 0$, pick $\delta = \epsilon > 0$

then for all x, $0 < |x - 0| < \delta \Rightarrow |x \sin\frac{1}{x} - 0| = |x \sin\frac{1}{x}| \le |x|$
$< \delta = \epsilon$

[redacted]

53. Let $f(x) = \begin{cases} \sin x & , x \neq \frac{\pi}{2} \\ 2 & , x = \frac{\pi}{2} \end{cases}$

Consider $x = \frac{\pi}{2}$, $f(x)$ increases to 1 as $x \to \frac{\pi}{2}^+$
or $x \to \frac{\pi}{2}^-$

So $f(x)$ gets closer to 2 , but $\lim_{x \to \frac{\pi}{2}} f(x) = 1$

57.(b)

For $\epsilon = \frac{1}{2}$, $\forall \delta > 0$, pick $x = 1 + \frac{\delta}{2}$

then $|x-1| = \frac{\delta}{2} < \delta$ but $|f(x) - 1| = |1 + \frac{\delta}{2} + 1 - 1|$
$= 1 + \frac{\delta}{2} > \frac{1}{2} = \epsilon$

[redacted]