Homework Assignment for Chapter 14

1. Section 14.2: Problems 41, 43, 49, 51, 57, 61, 63.

   Hint for problems 61, 63: Read 'Changing to Polar Coordinates' on page 781.

2. Section 14.3: Problems 19, 21, 53, 60, 65, 67, 69, 81, 91.

3. Show that if $f(x,y) = o(1)\cdot|x-x_0| + o(1)\cdot|y-y_0|$ as $(x,y)\to(x_0,y_0)$ then $f(x,y) = o(1)\cdot\sqrt{(x-x_0)^2+(y-y_0)^2}$ and vice versa (the converse). Note that all three $o(1)$ refer to $2D$ limits as $(x,y)\to(x_0,y_0)$.

   Hint: $\sqrt{\Delta x^2+\Delta y^2} = \frac{\Delta x}{\sqrt{\Delta x^2+\Delta y^2}}\Delta x + \frac{\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}\Delta y$

4. Section 14.4: Problems 7, 29, 31, 43, 51.

5. Suppose that $F(x,y,z)=0$ can implicitly define $x=f(y,z)$, or $y=g(z,x)$, or $z=h(x,y)$ near some point (x_0,y_0,z_0) with $F(x_0,y_0,z_0)=0$. (for example, $F(x,y,z) = x+2y+3z-4$ can). Show that, for any such point (x_0,y_0,z_0), we have

$$\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\frac{\partial h}{\partial x} = \frac{\partial f}{\partial z}\frac{\partial g}{\partial x}\frac{\partial h}{\partial y} = -1$$

6. Section 14.5: Problems 9, 15, 19, 25, 29, 35.

7. Let $f(x,y) = x^2y/(x^2+y^2)$ for $(x,y)\neq(0,0)$ and $f(0,0)=0$.

   (a) Is f continuous at $(0,0)$?

   (b) Do f_x and f_y exist at $(0,0)$?

   (c) Evaluate the directional derivative of f in the direction $(\cos\theta,\sin\theta)$, i.e. $df/ds_{(0,0),(\cos\theta,\sin\theta)}$, if it exists.

   (d) Is f differentiable at $(0,0)$?

8. Section 14.6: Problems 1, 9, 13, 19, 25, 33, 39, 45, 59, 63, 66, 67.

9. Section 14.7: Problems 1, 31, 35, 39, 43, 44, 49, 57.

10. Section 14.8: Problems 1, 23, 27, 33, 35.

11. Section 14.9: Problems 7, 9, 11.

12. Derive Taylor's formula for functions of three variables.

13. Section 14.10: Problems 3, 7, 9, 12.