Homework Assignment for Week 09

Assigned April 23, 2009.

1. Section 13.6: Problem 57.
2. Section 13.7: Problems 17, 35, 36, 37, 42, 45. Try to use the gradient analysis (i.e. sketch the gradient vectors to help you find the answer) in problem 37.
3. Section 13.8: Problems 7, 8, 25, 27, 29, 31. (Use the method of Lagrange Multipliers only).
4. Sketch the level curves and gradient vectors of $f(x, y) = x^2 \pm y^2$, respectively.
5. Suppose that $f_x(x, y) = 3x^2 + 2x + 2y$ and $f_y(x, y) = 2x + 2y$. Does f has a local max, local min or a saddle point at $(0, 0)$? Hint: try the gradient analysis.
6. Find the equation of plane normal to the curve

   $$\begin{cases} x^2 + 2y^2 + 3z^2 = 6 \\ x + y + z = 3 \end{cases}$$

   at $(1, 1, 1)$.
7. Suppose that the equation $F(x, y, z) = 0$ can define implicitly either $x = f(y, z)$, $y = g(z, x)$ or $z = h(x, y)$ ($F(x, y, z) = 2x + 3y - 4z - 1$ is such an example). Show that the following identity holds:

   $$f_y \cdot g_z \cdot h_x = -1.$$

   This identity is sometimes written as $x_y \cdot y_z \cdot z_x = -1$, which might be a little misleading and hard to understand.
8. The following identity

   $$\int_a^b \frac{d}{dy} f(x, y)dx = \frac{d}{dy} \int_a^b f(x, y)dx \tag{1}$$

   is valid provided the integrand is smooth enough. This is the case for most engineering applications including this homework problem.

   Use (1) and the Chain Rule to compute

   $$\frac{d}{dy} \int_1^2 \frac{\cos(xy)}{x} dx \quad \text{and} \quad \frac{d}{dy} \int_{1+y^2}^{2+\sin(y)} \frac{\cos(xy)}{x} dx$$