

Homework Assignment for Week 03

Assigned Mar 12, 2009.

1. Section 9.6: Problems 11, 15, 21, 23, 27, 28, 35, 39, 41, 44, 47.
2. Section 9.7: Problems 1, 3, 7, 15, 19, 25, 29, 33(show that equality holds), 35, 47(a), 50, 57, 58.
3. Find a power series whose interval of convergence is $[1, 3)$. Do the same for $(1, 3)$, $[1, 3]$ and $(1, 3]$, respectively.
4. (Challenge of the week)

Find the interval of convergence for the power series

$$1 + \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots + \frac{1 \cdots (2n-1)}{2 \cdots 2n}x^n + \cdots$$

Hint: Denote by $a_n = \frac{1 \cdots (2n-1)}{2 \cdots 2n}$ and define $b_2 = \frac{2}{3}$, $b_3 = \frac{2 \cdot 4}{3 \cdot 5}$, \cdots , $b_n = \frac{2 \cdots (2n-2)}{3 \cdots (2n-1)}$.

For $x = 1$, the fact that $b_n < 1$ will help.

For $x = -1$, compare a_n with b_n and $2b_n$ to conclude that $a_n \sim n^{-p}$. What is p ?

Do the same for

$$1 + \frac{1}{3}x + \frac{1 \cdot 4}{3 \cdot 6}x^2 + \frac{1 \cdot 4 \cdot 7}{3 \cdot 6 \cdot 9}x^3 + \cdots + \frac{1 \cdots (3n-2)}{3 \cdots 3n}x^n + \cdots$$