Homework Assignment for Week 08

1. Section 13.4: Problems 5, 7, 8, 13, 19, 21, 29, 23, 35, 36.

2. (s13.4-extra1)

   If $f(x, y)$ is continuous at $(0, 0)$ along each straight line and each parabola that passes through $(0, 0)$, i.e., if for each $a \in R$,

   $$\lim_{t\to 0} f(t, at) = \lim_{t\to 0} f(at, t) = \lim_{t\to 0} f(t, at^2) = \lim_{t\to 0} f(at^2, t) = f(0, 0).$$

   Does it mean that $\lim_{(x,y)\to(0,0)} f(x, y) = f(0, 0)$? Explain.

3. (s13-4 extra2)

   Let $f(x, y) = \sqrt{x^2 + 2y^2}$. Is $f(x, y)$ continuous at $(x, y) = (0, 0)$? Is it differentiable at $(0, 0)$?

4. (s13.4-extra3)

   Show that if $f(x, y) = o(1) \cdot |x - x_0| + o(1) \cdot |y - y_0|$ as $(x, y) \to (x_0, y_0)$ then $f(x, y) = o(1) \cdot \sqrt{(x - x_0)^2 + (y - y_0)^2}$ and vice versa (the converse). Note that all three $o(1)$ refer to $2D$ limits as $(x, y) \to (x_0, y_0)$.

5. Section 13.5: Problems 7, 11, 21, 23, 29, 31, 39, 43.

6. (s13.5-extra1)

   The following identity

   $$\int_a^b \frac{d}{dy} f(x, y)dx = \frac{d}{dy} \int_a^b f(x, y)dx \tag{1}$$

   is valid provided the integrand is smooth enough. For most engineering applications including this homework problem, the integrand is smooth enough and therefore (1) holds.

   Use (1) and the Chain Rule to compute

   $$\frac{d}{dy} \int_1^2 \frac{\cos(xy)}{x} dx \quad \text{and} \quad \frac{d}{dy} \int_{1+y^2}^{2+\sin(y)} \frac{\cos(xy)}{x} dx$$