Homework Assignment for Chap 02

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1. Section 2.2: problems 25, 55, 72(b).
2. Section 2.3: problems 23, 33, 40, 46.
3. State (need not prove) the '$x \to c^+$' and '$x \to \infty$' versions of the Sandwich Theorem. Part of the assumption in the standard Sandwich Theorem reads

   $$\cdots \text{for all } x \neq c \text{ in some open interval about } c \cdots$$

   How would you change this sentence in the '$x \to c^+$' and '$x \to \infty$' versions, respectively?
4. Section 2.4: problems 54, 55, 56, 57.
5. Chap 2: problems 41, 42, 53, 69, 70.
6. For those of you who are really not confident about your high school mathematics, pick some among section 2.3 problems 1-22, 35-40, section 2.4 problems 37-48 and practice yourself. Normally you don't need this.
7. Evaluate $\displaystyle\lim_{\theta \to \pi/6} \frac{\sin\theta - 1/2}{\theta - \pi/6}$
8. (Challenge problem, optional)

   Let $f : (0,1) \longrightarrow R$ be defined as

   $$f(x) = \begin{cases} 1/p & \text{if } x = q/p, \quad p, q \in N, \quad (p,q) = 1 \\ 0 & \text{otherwise} \end{cases}$$

   For what values of $c \in (0,1)$ is f continuous at c?
9. Section 2.5: problems 30, 32, 35, 38, 41.
10. Chap 2: problems 74, 75.
11. How would you define the following limits formally using ϵ and δ? (Need not prove anything, just define them)

    a.

    $$\lim_{x \to c^+} f(x) = L$$

b.

$$\lim_{x \to c} f(x) = \infty$$

c.

$$\lim_{x \to -\infty} f(x) = L$$

Hint: The formal definition of $\lim_{x \to c} f(x) = L$ is a translation of

$$f(x) \text{ can be arbitrarily close to } L \text{ as long as } x \neq c \text{ is close enough to } c$$

The '$f(x) \to \infty$' part, in plain words can be like '$f(x)$ be arbitrarily large' while the 'as $x \to \infty$' part can be 'whenever x is large enough'. The latter, in mathematical language, would be 'there is an M such that for all $x > M$, $\cdots$'

12. Use the $\epsilon - \delta$ argument to show that, if $f(x)$ and $g(x)$ are both continuous at $x = c$, then so is $f(x) + g(x)$ and $f(x) - g(x)$.

13. Use the $\epsilon - \delta$ argument to show that if $\lim_{x \to c} f(x) = L$ and $g(y)$ is continuous at $y = L$, then $\lim_{x \to c} g(f(x)) = g(L)$.

14. Use the $\epsilon - \delta$ argument to show that if $f(x)$ is continuous at $x = c$, then so is $3f(x)$.