Homework Assignment for Week 11

1. Section 6.6: Problems: 17, 21, 27, 33, 37-46, 51, 54, 55.

2. (s6.6-extra1) Cauchy's Mean Value Theorem
   Prove the following variant of the Mean Value Theorem:
   Suppose f and g are continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that
   $$\begin{vmatrix} f(b) - f(a) & f'(c) \\ g(b) - g(a) & g'(c) \end{vmatrix} = 0. \tag{1}$$
   Note that, this c need not satisfy $\dfrac{f(b) - f(a)}{b - a} = f'(c)$, nor $\dfrac{g(b) - g(a)}{b - a} = g'(c)$, but only $\dfrac{f(b) - f(a)}{g(b) - g(a)} = \dfrac{f'(c)}{g'(c)}$, which is the same as (1) provided $g(b) - g(a) \neq 0$.
   Hint: Apply standard Mean Value Theorem to
   $$F(x) = \begin{vmatrix} f(b) - f(a) & f(x) - f(a) \\ g(b) - g(a) & g(x) - g(a) \end{vmatrix} \qquad \text{on } [a, b].$$

3. (s6.6-extra2) Use Cauchy's Mean Value Theorem to prove the strong form of l'Hôpital's rule.

4. Section 6.7: Problems: 5, 7, 8, 11, 12, 17(a), 18.

5. Section 6.9: Problems: 9, 13, 21, 23, 31, 33, 39, 43, 57, 61, 67, 71, 79.

6. (s6.9-extra1) Show that
   $$\frac{d \csc^{-1} y}{dy} = \frac{-1}{|y|\sqrt{y^2 - 1}}, \qquad |y| > 1$$
   Explain why the negative sign is chosen. You will need to start from the 'restricted' domain of csc to see this.