Homework Assignment for Week 02

1. Section 2.5: problems 30, 32, 35, 38, 41.
2. Chap 2: problems 74, 75.
3. (s2.5-extra1) How would you define the following limits formally using ϵ and δ? (Need not prove anything, just define them)

   a.
   $$\lim_{x\to c^+} f(x) = L$$

   b.
   $$\lim_{x\to c} f(x) = \infty$$

   c.
   $$\lim_{x\to -\infty} f(x) = L$$

   Hint: The formal definition of $\lim_{x\to c} f(x) = L$ is a translation of

   $f(x)$ can be arbitrarily close to L as long as $x \neq c$ is close enough to c

   The '$f(x) \to \infty$' part, in plain words can be like '$f(x)$ be arbitrarily large' while the 'as $x \to \infty$' part can be 'whenever x is large enough'. The latter, in mathematical language, would be 'there is an M such that for all $x > M$, $\cdots$'

4. (s2.5-extra2) Use the $\epsilon - \delta$ argument to show that, if $f(x)$ and $g(x)$ are both continuous at $x = c$, then so is $f(x) + g(x)$ and $f(x) - g(x)$.
5. (s2.5-extra3) Use the $\epsilon - \delta$ argument to show that if $\lim_{x\to c} f(x) = L$, and $g(y)$ is continuous at $y = L$, then $\lim_{x\to c} g(f(x)) = g(L)$.
6. (s2.5-extra4. Challenge of the week, optional)
   Let $f : (0,1) \longrightarrow R$ be defined as

   $$f(x) = \begin{cases} 1/p & \text{if } x = q/p, \quad p, q \in N, \quad (p,q) = 1 \\ 0 & \text{otherwise} \end{cases}$$

   For what values of $c \in (0,1)$ is f continuous at c?