Midterm 1

April 01, 2014

Show all details.

- 1. (12 pts) Find the solution of $3xy' y = 1 + \ln x$ on x > 0 with y(1) = -2.
- 2. (16 pts) Show that $\sum_{k=1}^{\infty} k^{-2}$ converges and evaluate $\lim_{n \to \infty} \frac{\log\left(\sum_{k=n}^{\infty} k^{-2}\right)}{\log n}$. Give details. Hint: If the limit is p, this means that the sum in the limit is approximately n^p . Find p and prove it.
- 3. (10+10 pts)
 - (a) Show that the series $1 \frac{\pi^2}{4 \cdot 2!} + \frac{\pi^4}{16 \cdot 4!} \dots + (-1)^n \frac{\pi^{2n}}{2^{2n} \cdot (2n)!} + \dots$ converges *absolutely*. Explain.
 - (b) Find the sum of the series in (a). <u>Prove your answer</u>.Hint: it is related to the Taylor series of an elementary function.
- 4. (16 pts) Give an approximation of $\int_0^{\frac{1}{2}} \sin(x^2) dx$ to within 10^{-8} . Give the formula of the approximation, need not give the numerical value. Explain why the error is less than 10^{-8} .
- 5. (10+10 pts) True or False? Prove it if true, give a counter example if false.

(a) If
$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots$$
 on $|x| < 1$, then $a_n = f^{(n)}(0)/n!$.
(b) If $g(x) = f(0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ on $|x| < 1$, then $f(x) = g(x)$ on $|x| < 1$

6. (16 pts) Let the region R be given as 'inside of r = 1' and 'outside of $r = 1 - \cos \theta$ '. Find the area of R, the perimeter of R and surface area obtained from evolving R around the x-axis. Be careful about graphing R as it affects all that follows There are certain symmetry in the graph of R that you should notice.