Homework Assignment for Week 02 (revised again on Sep 25, 2011)

Assigned Sep 21, 2011.

1. Section 2.5: problems 30, 32, 35, 38, 41.
2. Chap 2: problems 74, 75.
3. How would you define the following limits formally using ϵ and δ? (Need not prove anything, just define them)

   a.
   $$\lim_{x\to c^+} f(x) = L$$

   b.
   $$\lim_{x\to c} f(x) = \infty$$

   c.
   $$\lim_{x\to -\infty} f(x) = L$$

   d.
   $$\lim_{x\to \infty} f(x) = -\infty$$

   Hint: The formal definition of $\lim_{x\to c} f(x) = L$ is a translation of

   $f(x)$ can be arbitrarily close to L as long as $x \neq c$ is close enough to c

   The '$f(x) \to \infty$' part, in plain words can be like '$f(x)$ be arbitrarily large' while the 'as $x \to \infty$' part can be 'whenever x is large enough'. The latter, in mathematical language, would be 'there is an M such that for all $x > M$, $\cdots$'

4. Use the $\epsilon - \delta$ argument to show that, if $f(x)$, $g(x)$ and $h(x)$ are all continuous at $x = c$, then so is $f(x) + g(x) - 2h(x)$.
5. Use the $\epsilon - \delta$ argument to show that if $\lim\limits_{x\to c} f(x) = L$, and $g(y)$ is continuous at $y = L$, then $\lim\limits_{x\to c} g(f(x)) = g(L)$.
6. Use the $\epsilon - \delta$ argument to show that if $f(x)$ and $g(x)$ are both continuous at $x = c$, and $|f(x)| \leq 5$ for all x, then $f(x)g(x)$ is also continuous at $x = c$.
7. Use the $\epsilon - \delta$ argument to disprove that $\lim\limits_{x\to 0^-} x^2 = 1$.
8. Section 3.1: problems 35, 40, 49.