Final Exam

Jan 04, 2012

1. (12 pts) Find the solutions for $\dfrac{dy}{dx} = e^{x-y}$ and $x\dfrac{dy}{dx} + y = \sin x$, respectively.

2. (12 pts) Find the volume and surface area of the object obtained by rotating the region $(x-2)^2 + y^2 \le 1$ around the y axis (ie. a donut).

3. (6 pts) Give the partial fraction expansion of $\dfrac{x^{10}}{(1-x^4)^2}$. Need not find any of the undetermined coefficients.

4. (48 pts) Evaluate

   (a) $\displaystyle\int \frac{1}{2+\sin x}\,dx$ (b) $\displaystyle\int x^x(1+\ln x)\,dx$ (c) $\displaystyle\int \frac{1}{\sqrt{4x-x^2}}\,dx$

   (d) $\displaystyle\int_1^2 \frac{1}{e^x - e^{-x}}\,dx$ (e) $\displaystyle\int_{-1}^1 \frac{1}{x^2+2x+2}\,dx$ (f) $\displaystyle\int_0^{\pi/2} \sin^4 x \cos^3 x \,dx$

5. (12 pts) Prove that the improper integral $\displaystyle\int_0^\infty x^2 e^{-x}\,dx$ converges. Then find its value.

6. (8 pts) Does the improper integral $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3-x}}\,dx$ converge or diverge? Explain (DO NOT try to integrate explicitly).

7. (12 pts) Evaluate $\displaystyle\int_0^\pi \sqrt{1+\sin x}\,dx$