Homework Assignment for Week 02

Assigned Sep 21, 2006.

1. Section 2.5: problems 30, 32, 35, 38. Chap 2: problems 74, 75.
2. How would you define the following limits formally using ϵ and δ?

   a.
   $$\lim_{x\to c^+} f(x) = L$$
   b.
   $$\lim_{x\to c} f(x) = \infty$$
   c.
   $$\lim_{x\to -\infty} f(x) = L$$

   Remark: The formal definition of $\lim_{x\to c} f(x) = L$ is a translation of

   $f(x)$ can be arbitrarily close to L as long as $x \neq c$ is close enough to c

   The '$f(x) \to \infty$' part, in plain words can be like '$f(x)$ be arbitrarily large' while the 'as $x \to \infty$' part can be 'whenever x is large enough'. The latter, in mathematical language, would be 'there is a M such that for all $x > M$, $\cdots$'
3. Use the $\epsilon - \delta$ argument to show that, if $f(x)$ and $g(x)$ are both continuous at $x = c$, then so is $f(x) + g(x)$ and $f(x) - g(x)$.
4. Use the $\epsilon-\delta$ argument to show that if $f(x)$ is continuous at $x = c$ and $g(y)$ is continuous at $y = f(c)$, then $g(f(x))$ is continuous at $x = c$.
5. Is the following statement true or false?

   If the function $y = f(x)$ defined on $[a, b]$ takes any value between $f(a)$ and $f(b)$, then $f(x)$ is continuous on $[a, b]$.
6. Read 3.1: Examples 6, 8, 10. Section 3.1: problems 35, 40, 49.
7. (Challenge of the week, optional. These problems are for fun only and will not appear in any exams. Normally we will not have time to discuss it in the recitation session, either. Enjoy the challenge.)

   Let $f : (0, 1) \longrightarrow R$ be defined as
   $$f(x) = \begin{cases} 1/p & \text{if } x = q/p, \quad p, q \in N, \quad (p, q) = 1 \\ 0 & \text{otherwise} \end{cases}$$
   For what values of $c \in (0, 1)$ is f continuous at c?

8. (Challenge of last week, also optional) How does one factorize a cubic polynomial? Someone brought this up after I made up homework 01, so here you go.

   (a) For cubic polynomials without the quadratic term $x^3 + bx + c$, one can make use of the factorization

$$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)(\cdots)$$

   (b) For general cubic polynomials x^3+ax^2+bx+c, make a change of variable $y = x+\cdots$ to reduce it to the special case above.