Homework Assignment 3.

Given Mar 11, due Mar 23.

1. Section 9.8: 10, 14, 28, 29, 31, 32, 33, 34(a).
2. Chap 9: 27, 28, 29, 30, 32, 34, 49-54.
3. Read p 619, which says the Taylor series generated by $(1+x)^m$, $m \in R$, converges on $|x| < 1$. It is not clear the series converges to $(1+x)^m$ itself since analyzing $R_n(x)$ directly is not quite easy (try it and you'll see why). An alternative approach is through the following steps:

   (a) Verify that
   $$(k+1)\begin{pmatrix} m \\ k+1 \end{pmatrix} + k\begin{pmatrix} m \\ k \end{pmatrix} = m\begin{pmatrix} m \\ k \end{pmatrix}$$

   (b) Define, for $|x| < 1$,
   $$f(x) = \sum_{k=0}^{\infty} \begin{pmatrix} m \\ k \end{pmatrix} x^k = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots.$$
   Show that $f(0) = 1$ and
   $$(1+x)f'(x) = mf(x)$$
   on $|x| < 1$.

   (c) Show that $f(x) = (1+x)^m$.