

Homework Assignment 3.

Given Mar 11, due Mar 23.

1. Section 9.8: 10, 14, 28, 29, 31, 32, 33, 34(a).
2. Chap 9: 27, 28, 29, 30, 32, 34, 49-54.
3. Read p 619, which says the Taylor series generated by $(1+x)^m$, $m \in \mathbb{R}$, converges on $|x| < 1$. It is not clear the series converges to $(1+x)^m$ itself since analyzing $R_n(x)$ directly is not quite easy (try it and you'll see why). An alternative approach is through the following steps:

(a) Verify that

$$(k+1) \binom{m}{k+1} + k \binom{m}{k} = m \binom{m}{k}$$

(b) Define, for $|x| < 1$,

$$f(x) = \sum_{k=0}^{\infty} \binom{m}{k} x^k = 1 + mx + \frac{m(m-1)}{2!} x^2 + \frac{m(m-1)(m-2)}{3!} x^3 + \dots$$

Show that $f(0) = 1$ and

$$(1+x)f'(x) = mf(x)$$

on $|x| < 1$.

(c) Show that $f(x) = (1+x)^m$.