Final Exam

Jan 13 2005, 10:10 AM
Show all your work

1. (10 pts) Compute $\frac{d}{dx}\int_x^{x^2} \sin y\, dy$.
2. Evaluate (10 pts)
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2} \sin^2(\frac{i}{n})$$
3. (4pts + 6 pts) Suppose $f(1) = f(2) = 0$ and $f(x) > 0$ on $1 < x < 2$ with $f(x)$ and $f'(x)$ continuous. Write down the formula for the following quantities (formula only, no numerical values required).
   (a) The area and perimeter of the region enclosed by $y = f(x)$ and the x axis.
   (b) The volume and surface area obtained by rotating the graph of f around the x axis.
4. (8 pts each) Which of the following improper integrals converge and which of them diverge? Explain in detail. (Do not try to integrate explicitly)
   (a) $\int_1^3 \frac{1}{\sqrt{x^3-4x}} dx$
   (b) $\int_1^\infty \frac{\ln x}{x^2} dx$
5. (6 pts each) Solve the following ODE's:
   (a) $t\frac{dy}{dt} + 3y = t, t > 0 \qquad y(1) = 1$
   (b) $\frac{dy}{dt} = e^{y-2t}, \qquad y(0) = 0$
6. (6 pts each) Find the following integrals.
   (a) $\int \frac{1}{e^x - e^{-x}} dx$
   (b) $\int \frac{1}{2+\sin x} dx$
   (c) $\int \frac{3+x^2}{1-x^4} dx$
   (d) $\int_0^1 xe^{-x} dx$
   (e) $\int_0^1 \frac{1}{(1+x^2)^2} dx$
7. (4 pts each) State (need not prove) integral test, ratio test and root test for positive series.