1 (a) $\int \frac{\tan^{-1}x}{x^2}\,dx = \int \tan^{-1}x\; d\left(-\frac{1}{x}\right) = -\frac{\tan^{-1}x}{x} - \int -\frac{1}{x}\, d(\tan^{-1}x)$

$= -\frac{\tan^{-1}x}{x} + \int \frac{1}{x}\frac{1}{1+x^2}\,dx = -\frac{\tan^{-1}x}{x} + \int \left(\frac{1}{x} - \frac{x}{1+x^2}\right)dx$

$= -\frac{\tan^{-1}x}{x} + \ln|x| - \frac{1}{2}\ln(1+x^2) + C$

(b) $\int \frac{\cos x\,dx}{1+\sin^2 x} = \int \frac{d\sin x}{1+\sin^2 x} = \tan^{-1}(\sin x) + C$

(c) $\int \frac{dx}{x^2\sqrt{1+x^2}} = \int \frac{\sec^2\theta\, d\theta}{\tan^2\theta\,|\sec\theta|} = \int \frac{\sec\theta}{\tan^2\theta}d\theta = \int \frac{\cos\theta}{\sin^2\theta}d\theta = -\frac{1}{\sin\theta} + C$

$(x = \tan\theta,\ dx = \sec^2\theta\, d\theta,\ \theta \in (-\tfrac{\pi}{2}, \tfrac{\pi}{2}))$ $= -\frac{\sqrt{1+x^2}}{x} + C$

~~$x \in (-\frac{\pi}{2},$~~

(d) $\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx = \int \frac{x^2}{(x-1)(x+1)^2}\,dx$

Find A, B C. such that

$\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B(x+1)+C}{(x+1)^2}$ $\Rightarrow$ Ans.

$\Rightarrow$ ~~(2)~~ $\begin{cases} A+B=1 \\ 2A+C=0 \\ A-B-C=0 \end{cases} \Rightarrow A=\frac{1}{4},\ B=\frac{3}{4},\ C=-\frac{1}{2}$

$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2}(x+1)^{-1} + C$

2 (a) $\int_2^\infty \frac{1}{\ln x}\,dx > \int_2^\infty \frac{1}{x}\,dx = \infty$ $\therefore$ diverge.

(b) $\int_{-1}^{1} x\ln|x|\,dx$ possible singularity: $x=0$

$\int_0^1 x\ln x\,dx$: $\lim_{x\to 0}\left|\frac{x^{\frac{1}{2}}\ln x}{\sqrt{x}}\right| = \lim_{x\to 0} x^{\frac{1}{2}}\ln x = 0$

and $\int_0^1 x^{\frac{1}{2}}$ converges $\therefore$ $\int_0^1 x\ln x\,dx$ converges

Similar for $\int_{-1}^0 x\ln|x|\,dx$.