Final Exam

June 16, 2004, 10:10 AM. SHOW ALL YOUR WORK.

1. (15 pts) Use the method of Lagrangian multipliers (and NOT any other methods) to find the extreme values of the function $f(x,y,z) = xy + z^2$ on the circle in which the plane $y - 2x = 0$ intersects the sphere $x^2 + y^2 + z^2 = 1$.

2. (10 pts) Switch the order of integration of

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dzdydx$$

   to $dydxdz$ and $dxdydz$ respectively. You don't need to find the numerical value of the integral.

3. (15 pts) Let R be the region in the first quadrant bounded by $xy = 1$, $xy = 4$ and the lines $y = x$, $y = \frac{x}{4}$. Compute $\int\int_R (\sqrt{\frac{y}{x}} + \sqrt{xy}) dxdy$.

4. (15 pts) Find the integral of $G(x,y,z) = x$ over the surface given by $\{y = x^2,\ 0 \le x \le 2,\ 0 \le z \le 3\}$

5. (15 pts) Let C be the upper half of the ellipse $\frac{x^2}{4} + y^2 = 1$, $y \ge 0$, counterclockwise oriented. $\boldsymbol{F}(x,y) = (-y, 3x)$. Compute $\int_C \boldsymbol{F} \cdot \boldsymbol{T} ds$.

6. (30 pts) Let $D = \{x^2 + y^2 + z^2 \le 1, z \ge 0\}$, $S = \{x^2 + y^2 + z^2 = 1, z \ge 0\}$. Verify the Stokes Theorem and divergence Theorem for $\boldsymbol{F} = (x+z, xz, xy)$ on S and D (Compute integrals on both sides and check they are the same).