HW11

Textbook §4.4 #7.

Run the following code, then you will get the results.

- (a) 3.15947567425982
- (b) 3.10933712650887
- (c) 3.00906003100695

```
format long
a = 0;
b = 2;
h = 0.25;

%Composite Trapezoidal
n = (b-a)/h;
x = a:h:b;
f = x.^2.*log(x.^2+1);
w = 2*ones(1,n+1);
w(1) = w(n+1) = 1;
s = w*f'*h/2

%Composite Simpson's
n = (b-a)/h;
x = a:h:b;
f = x.^2.*log(x.^2+1);
w = ones(1,n+1);
w(2:2:n  ) = 4;
w(3:2:n-1) = 2;
s = w*f'*h/3

%Composite Midpoint
n = (b-a)/h-2;
x = a+h:2*h:b-h;
f = x.^2.*log(x.^2+1);
w = ones(1,n/2+1);
s = w*f'*2*h
```

Textbook §4.4 #14.

- (a) $|\frac{b-a}{12}h^2 f''(\mu)| = \frac{1}{12}h^2\frac{1}{\mu} \le \frac{h^2}{12} < 10^{-5} \Rightarrow h < \sqrt{12 \times 10^{-5}} \approx 0.01095$. And, $n = \frac{b-a}{h} = \frac{1}{h} > \frac{1}{\sqrt{12\times10^{-5}}} \approx 91.28709 \Rightarrow n = 92$. Run the code to find the approximation 0.636301185569557.

- (b) $|\frac{b-a}{180}h^4 f^{(4)}(\mu)| = \frac{1}{180}h^4\frac{2}{\mu^3} \le \frac{h^4}{90} < 10^{-5} \Rightarrow h < \sqrt[4]{9 \times 10^{-4}} \approx 0.173205$. And, $n = \frac{b-a}{h} = \frac{1}{h} > \frac{1}{\sqrt[4]{9\times10^{-4}}} \approx 5.77350 \Rightarrow n = 6$. Run the code to find the approximation 0.636297500790914.

- (c) $|\frac{b-a}{6}h^2 f''(\mu)| = \frac{1}{6}h^2\frac{1}{\mu} \le \frac{h^2}{6} < 10^{-5} \Rightarrow h < \sqrt{6 \times 10^{-5}} \approx 0.0077460$. And, $n = \frac{b-a}{h} - 2 = \frac{1}{h} - 2 > \frac{1}{\sqrt{6\times10^{-5}}} - 2 \approx 127.09944 \Rightarrow n = 128$. Run the code to find the approximation 0.636287525399937.

```
format long
a = 1;
b = 2;

%Composite Trapezoidal
n = 92;
h = (b-a)/n;
x = a:h:b;
f = x.*log(x);
w = 2*ones(1,n+1);
w(1) = w(n+1) = 1;
s = w*f'*h/2

%Composite Simpson’s
n = 6;
h = (b-a)/n;
x = a:h:b;
f = x.*log(x);
w = ones(1,n+1);
w(2:2:n  ) = 4;
w(3:2:n-1) = 2;
s = w*f'*h/3

%Composite Midpoint
n = 128;
h = (b-a)/(n+2);
x = a+h:2*h:b-h;
f = x.*log(x);
w = ones(1,n/2+1);
s = w*f'*2*h
```

Textbook §4.4 #23.

To show that the sum

$$\sum_{j=1}^{n/2} f^{(4)}(\xi_j)2h$$

is a Riemann Sum, let $y_j = x_{2j}$, for $j = 0, \dots, n/2$. Then $\Delta y_j = y_j - y_{j-1} = 2h$ and $y_{j-1} < \xi_j < y_j$, for $j = 1, \dots, n/2$. Thus,

$$\sum_{j=1}^{n/2} f^{(4)}(\xi_j)\Delta y_j = \sum_{j=1}^{n/2} f^{(4)}(\xi_j)2h$$

is a Riemann Sum for $\int_a^b f^{(4)}(x)\,dx$. Hence

$$E(f) = -\frac{h^5}{90}\sum_{j=1}^{n/2} f^{(4)}(\xi_j) = -\frac{h^4}{180}\left[\sum_{j=1}^{n/2} f^{(4)}(\xi_j)2h\right] \approx -\frac{h^4}{180}\int_a^b f^{(4)}(x)\,dx = -\frac{h^4}{180}[f'''(b) - f'''(a)].$$

Textbook §4.4 #24.

The details are similar to #23 and left as an exercise.

- (a)

$$E(f) \approx -\frac{h^2}{12}[f'(b) - f'(a)].$$

- (b)

$$E(f) \approx \frac{h^2}{6}[f'(b) - f'(a)].$$

Textbook §4.4 #26.

- (a) Composite Trapezoidal rule:

$$E(f) \approx -\frac{h^2}{12}\ln 2 \approx -6.926 \times 10^{-6} \text{ as } h = 0.01095.$$

- (b) Composite Simpson's rule:

$$E(f) \approx -\frac{h^4}{180}[-\frac{1}{4} + 1] \approx -3.75 \times 10^{-6} \text{ as } h = 0.173205.$$

- (c) Composite Midpoint rule:

$$E(f) \approx \frac{h^2}{6}\ln 2 \approx 6.932 \times 10^{-6} \text{ as } h = 0.007746.$$

Textbook §4.7 #2b.

Use Table 4.12 to compute

$$\int_a^b f(x)\,dx = c_{2,1}g(r_{2,1}) + c_{2,2}g(r_{2,2}) \approx -0.730723036274041$$

where $g(t) := f\left(\frac{(b-a)t+(b+a)}{2}\right)\frac{b-a}{2}$.

The exact value is

$$\ln\frac{1.44}{3} \approx -0.733969175080200.$$

Textbook §4.7 #4b.

Use Table 4.12 to compute

$$\int_a^b f(x)\,dx = c_{3,1}g(r_{3,1}) + c_{3,2}g(r_{3,2}) + c_{3,3}g(r_{3,3}) \approx -0.733799022287857$$

where $g(t) := f\left(\frac{(b-a)t+(b+a)}{2}\right)\frac{b-a}{2}$.

The exact value is

$$\ln\frac{1.44}{3} \approx -0.733969175080200.$$

Textbook §4.7 #11.

Let $f(x) = x^n$ for $n = 0, 1, 2, 3$. Then solve

$$\begin{aligned} a + b &= 2 \\ -a + b + c + d &= 0 \\ a + b - 2c + 2d &= 2/3 \\ -a + b + 3c + 3d &= 0 \end{aligned}$$

to find the solution $a = 1,\ b = 1,\ c = 1/3$, and $d = -1/3$.

Check that the formula doesn't hold for $f(x) = x^4$. (Check!)

Textbook §4.7 #13.

- For $n = 2$, $P_2(x) = x^2 - \frac{1}{3}$

$$\Rightarrow r_{2,1} = \frac{1}{\sqrt{3}}, \quad r_{2,2} = -\frac{1}{\sqrt{3}}.$$

  Furthermore,

$$c_{2,1} = \int_{-1}^{1} \frac{x - (-\frac{1}{\sqrt{3}})}{\frac{1}{\sqrt{3}} - (-\frac{1}{\sqrt{3}})}\, dx = 1$$

$$c_{2,2} = \int_{-1}^{1} \frac{x - \frac{1}{\sqrt{3}}}{(-\frac{1}{\sqrt{3}}) - \frac{1}{\sqrt{3}}}\, dx = 1.$$

- For $n = 3$, $P_3(x) = x^3 - \frac{3}{5}x$

$$\Rightarrow r_{3,1} = \sqrt{\frac{3}{5}}, \quad r_{3,2} = 0, \quad r_{3,3} = -\sqrt{\frac{3}{5}}.$$

  Furthermore, (Exercise!)

$$c_{3,1} = \frac{5}{9}, \quad c_{3,2} = \frac{8}{9}, \quad c_{3,3} = \frac{5}{9}.$$

Textbook §4.7 #14.

Let $P(x) = \prod_{i=1}^{n} (x - x_i)^2$. Then $Q(P) = 0$ and $\int_{-1}^{1} P(x)\, dx \overset{\text{(Why?)}}{>} 0$.