Homework Assignment for Week 11

1. Section 6.1: Problems 10, 15(a), 16(a).

   For problems 15(a) and 16(a), just count multiplications/divisions.

2. Derive a linear system of equation corresponding to the following boundary value problem
$$u''(x) + u'(x) = f(x), \quad x \in (0,1)$$
$$u(0) = \alpha, \ u(1) = \beta$$
   with uniformly spaced grids $0 = x_0 < x_1 < \cdots < x_N = 1$, $x_i - x_{i-1} = h = 1/N$, using second order finite difference method. That is, given α, β, and f_i, $i = 1, 2, \cdots, N-1$, try to derive a linear system of equations to solve for u_i, $i = 1, 2, \cdots, N-1$. You don't have to solve the linear system this week (but soon will).

3. Consider an $N^2 \times N^2$ matrix A with $a_{ij} = 0$ except for $i - j = 0, \pm 1, \pm 2$ (only five diagonals have nonzero entries). Estimate total number of multiplication needed for Gaussian elimination (without pivoting) on A. Give the leading order of the number of multiplication as KN^p. Find K and p.

4. Do the same for an $N^2 \times N^2$ matrix B with $b_{ij} = 0$ except for $|i - j| \leq N$ (only $2N+1$ diagonals have nonzero entries).

5. Do the same for an $N^2 \times N^2$ matrix C with $c_{ij} = 0$ except for $i - j = 0, \pm 1$ and $\pm N$ (only 5 separated diagonals have nonzero entries).

6. Section 6.2: Problems 1(a), 3(a), 5(a), 31 (all in paper-and-pencil instead of programming).