Homework Assignment for Week 05

1. Section 2.4: Problem 13. Also derive this formula by way of quadratic approximation of $x = f^{-1}(y)$ at $(y, x) = (f(x_n), x_n)$.

2. Use the code for Algorithm 2.2 to check if the standard fixed point iteration $x_{n+1} = g(x_n) = 2\cos(x_n)$ converge to root of $f(x) = x - 2\cos(x) = 0$. If not, can you explain why? Can you find a constant β such that the modified fixed point iteration $x_{n+1} = \beta x_n + (1-\beta)g(x_n)$ converges (at least locally)?

3. Section 2.5: Problems 12(a), 14, 15, 16.

4. In Example 1 of section 2.5, the condition in Theorem 2.14 is not satisfied. Nevertheless, one could still get faster convergence of $\hat{p}_n$ than p_n, but of the same order. Analyze Aitken's Δ^2 method to evaluate $\lim_{n\to\infty} \dfrac{\hat{p}_n - p}{p_{n+2} - p}$. Then verify your result numerically.