

Homework Assignment for Week 17

1. Section 8.1: Problems 2, 14.

2. Derive the continuous version of least square problem:

Give n and $f(x) : [0, 1] \mapsto R$, find $a_0, \dots, a_n$ to minimize the quantity

$$\int_0^1 (f(x) - (a_0 + a_1x + \dots + a_nx^n))^2 dx$$

Derive the normal equation for the coefficient vector $(a_0, \dots, a_n)$.

Remark: The matrix corresponding to this linear system is ill-conditioned for large n (why?). The discrete counter part, problem 14, is similarly ill-conditioned for large n . The proper way of solving these problems numerically for large n , say $n > 5$, is to incorporate the Gram-Schmidt procedure to select an orthogonal basis of polynomial functions in place of $1, x, x^2, \dots$. See section 8.2 if you are interested to find out the details.