

Homework Assignment for Week 07

Goal: Study both theoretical and practical aspects of Lagrangian interpolating polynomials.

1. Section 3.1: Problem 6(a). Write a program to evaluate the degree three polynomial using for loops.

Hint: One can compute

$$L_{n,k}(x) = \prod_{\substack{i=0, \\ i \neq k}}^n \frac{(x - x_i)}{(x_k - x_i)}$$

with a for loop in i for each of the $k = 0, \dots, n$, then evaluate

$$P(x) = \sum_{k=0}^n f(x_k) L_{n,k}(x)$$

with another for loop on k . Note that, in C, the index for an array $a(i)$ starts with $i = 0$ by default. However, the index starts with $i = 1$ in matlab. One should shift the index in the Lagrange interpolation formula accordingly.

Remark: This is not the most efficient method to evaluate the Lagrangian interpolation. The proper way is to evaluate $P(x)$ using Neville's method in section 3.2. An alternative way is to compute the coefficients of $P(x)$ using Newton's divided-difference formula in section 3.3, then evaluate $P(x)$.

2. Section 3.1: Problems 9, 10, 13(a), 17 (the last sentence simply means $h = (10 - 1)/n$ for some positive integer n).
3. Let $x_0, \dots, x_n$ be uniformly spaced nodes on $[a, b]$ with $x_j = a + jh$, $h = (b - a)/n$.
 - (a) Show that $|(x - x_0) \cdots (x - x_n)| \leq n!h^{n+1}$ on $a \leq x \leq b$.
Hint: Can you find an upper bound for $\max_{0 \leq i \leq n, x_0 \leq x \leq x_n} |x - x_i|$?
 - (b) Let P_n be the degree n interpolating polynomial of e^x with uniformly spaced nodes on $[0, 1]$. Show that

$$\max_{0 \leq x \leq 1} |e^x - P_n(x)| \leq Ch^n \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

Note that uniform convergence of interpolating polynomials as in (b) does not hold in general. The Illustration on p154-p157 is a good example. In general, we only have

$$\max_{a \leq x \leq b} |f(x) - P_n(x)| \leq C_n h^n$$

from Theorem 3.3. A well known example with similar behavior is $f(x) = 1/(1 + x^2)$ on $[-5, 5]$ with uniformly spaced nodes.