Homework Assignment for Week 06

Goal: Study both theoretical and practical aspects of Fixed Point Iteration and Newton's method for nonlinear system of equations.

1. Section 10.1:

   Suppose that $D \subset R^n$ and $G : D \mapsto D$ is sufficiently smooth.

   Denote by the matrix norm $|||A|||_2 = \sup_{x \neq 0} \dfrac{\|Ax\|_2}{\|x\|_2}$ for any $n \times n$ real matrix A, where $\|x\|_2$ is the standard ℓ_2 norm for $x \in R^n$.

   (a) Give a sufficient condition for uniqueness of the fixed point $x = G(x)$ in terms of $|||G'|||_2$.

   (b) Give a sufficient condition for convergence of the fixed point iteration $x^{k+1} = G(x^k)$ and its error estimate in terms of $|||G'|||_2$.

   Hint: Go through and mimic the proofs of Theorem 2.3, 2.4 and Corollary 2.5.

2. Section 10.1: Problems 3(a,b), 4(a,b).

3. Section 10.1:

   As in the scalar case, where knowing the approximate value of $f'(x_*)$ would help to design a fixed point iteration for solving the equation $f(x) = 0$ by finding a suitable equivalent form $x = g(x)$ by introducing a free parameter α. Try apply this technique to the following system of equations

$$\begin{aligned} 1x_1 + 2x_2 + 0.03 * \sin(x_1 + x_2) &= 4 \\ 5x_1 + 6x_2 + 0.07 * \cos(x_1 - x_2) &= 8 \end{aligned}$$

   and find a convergent fixed point iteration. If the equivalent form is not obvious to you, to use the trick as in the scalar case, but take α to be a 2×2 matrix.

   If you already did it on Tuesday's recitation, change the numbers and practice again yourself.

4. Section 10.2: Problems 7(a,b).