Numerical Analysis I, Fall 2020 (http://www.math.nthu.edu.tw/~wangwc/)

Quiz 01

Sep 29, 2020.

- 1. How many bits does it take to store a binary floating point number of the form $\pm 1.a_1a_2\cdots a_t \times 2^e$ with $t = 10, a_j \in \{0, 1\}, -14 \leq e \leq 15$? What is the distance from 1.0 to nearest floating number? Explain.
- 2. Suppose that if fl(y) is a k-digit rounding approximation to y. Show that

$$\left|\frac{y - fl(y)}{y}\right| \le 5 \times 10^{-k}$$

Remark: A k-digit rounding means, given $y = \pm 0.d_1d_2\cdots d_kd_{k+1}\cdots \times 10^n$, $0 \le d_i \le 9$, then fl(y) obtained by changing d_k to \tilde{d}_k according to the value of d_{k+1} .

- 3. We showed in class the estimate of relative error resulted from $x \times y$ in terms of ε_M . Derive the corresponding result for $x/y, y \neq 0$.
- 4. Solve for $x^2 2100x + 1 = 0$ to 15 correct digits using standard double precision arithmetic. Explain how you find your answer (No explanation, no points).
- 5. Let $A = \{(x 100)^2 + y^2 < (40\pi)^2\}$, and $B = \{x + y > 50.1\}$. Find the number of grid points (i, j) (i, j are integers) in $A \cap B$. Write down the answer and name your code by your student ID number.

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